The Long-run Relationship among Index-linked Bonds and Conventional Bonds

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Abstract: This paper investigates the existence of arbitrage between index linked bonds and conventional bonds. The long-run equilibrium relationship among two French bonds yields (The OAT yields and OATi) is also studied empirically. In practice, the Johansen methodology is applied to estimate different VAR-based cointegration tests. The presence of a structural break due to the subprime crisis is discussed as well.

Two main results are reached: First, the cointegration test indicates the existence of a long run relationship between the OAT and OATi return, and then the co-movement of the latter is confirmed, a result which is believed to be important to understand the bond market’s mechanism. Second, the structural break test shows the presence of structural change in the relation between the two types of bonds. This structural break is due to an increase of volatility in the OAT and OATi returns in the subprime crisis period.

Keywords: Index linked bonds, Cointegration, Structural break, Subprime crisis
JEL Classifications: C22, C8; E44; G1

1. Introduction

To begin, we can say that correlation and co-movement structure of stocks, bonds and other asset classes have important ramifications for investors looking to diversify their portfolios. In fact, most rational models of portfolio choice suggest that investors hold diversified portfolios to reduce or eliminate non compensated risk. We notice that investors’ diversification choices correlate with their individual characteristics. Indeed, adverse risk investors tend to invest more in bonds than in stocks to minimize the risk of their portfolios.

For instance, bond markets allow lenders to invest in relatively low risk assets and borrowers to obtain funds in relatively liquid markets. Different types of bonds exist in the market such as conventional bonds, convertible bonds and index linked bonds. It is important to understand that the main merit of the index linked bond comparing to the conventional one is that it reduces the risk by eliminating a part of inflation risk.

Besides, Inflation-linked bond markets have experienced significant growth in recent years, particularly in France. In this country, the first bond whose coupon payments were indexed to the French consumer price index (CPI) excluding tobacco (OATi)1 in 1998. Investors in OATi were initially mainly domestic, but later on the availability of inflation protection also attracted other investors.

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1 OATi : Obligations Assimilables du Trésor indexées.
euro area investors who were willing to accept the mismatch between French and their domestic inflation\(^2\). As a consequence, the gap between supply and demand for inflation-linked bonds in the euro area market was significant. Hence, to reduce this difference, French Treasury decided to issue a new ten-year bond indexed. Furthermore, French Treasury issued in 2001 first bond whose coupon payments were indexed to euro area inflation. Especially that since 2003, inflation-linked bonds market in the euro area has experienced significant growth.

Despite the importance of the index linked bond, there has been much less academic attention devoted to index linked bond markets than to conventional bonds or equity markets. The big gap is in empirical work, and the main reason for this is data availability. As a consequence, there are only a few studies on the index linked bond market; most of these are manifested on US bond market. In this field, Dryden and Hancock (1992) studied the index linked bond duration on Australian bonds. In another field, Deacon and Derry (1994) discussed various methods by which prices of index linked bonds can be compared with prices of conventional bonds. Also we observe that Shen (1995) presented the benefits and limitations of inflation Indexed Treasury Bonds, whereas Barone and Maseras (1996) presented the merit of index linked bonds and the importance of such financial asset. Besides, Campbell and Shiller (1996) studied the inflation risk premium. More recently, Reschreiter (2006) implemented autoregressive (VAR) model to investigate the impact of revisions in inflation expectations on the prices of UK inflation-indexed bonds. Last but not least Breedon (2012) undertook a variance decomposition of index linked bond issued by the government of Iceland, the united state and the United Kingdom.

The contributions of this study are the following. First, we examine the existence of arbitrage between index linked bonds and conventional ones. Second, we estimate a VAR-based cointegration test using the methodology developed in Johansen (1991) to examine the long-run equilibrium relationship among two French bonds yields (The OAT and OAT\(i\)). The temporal stability of the both returns relationship is also discussed. To this aim we examine the relationship between the two bonds over three periods of time (pre, post and during the subprime crisis).

This paper is structured as follows. Section 2 presents the literature and theoretical back grounds. Then, in section 3 the data and summary statistics are showed. After that, section 4 discusses the empirical results, and section 5 concludes this paper.

2. Literature and Theoretical Backgrounds

2.1 Arbitration of Index-linked bond and Conventional Bond

A market of bonds so important should operate well. For the bond market, this means both efficiency and liquidity. Efficient bond prices, incorporating all available information, will be better signals to investors and savers than if the markets did not incorporate fully relevant information into prices.

Given the assumed market efficiency in bonds and index linked bond no arbitrage relationship is expected from this two assets.

The formulation of such idea to the index linked bond (OAT\(i\)) and conventional bond (OAT) can be explained as:

\[
E(OAT, / F_t) = E(OAT_i, ) + Irp_t + Lrp_t
\]

where \(Ir_p\) denotes inflation risk premium, and \(Lrp_t\) denotes liquidity risk premium.

\(^2\)Garcia A. Van Rixtel A (2007), Inflation linked bonds from a central bank perspective.
The expected return on conventional bonds given the set of information \( F_t \) depends on that of index linked bond.

This modeling suppose that the Information set \( F_t \) is not constant but it is likely to be varying given the dynamic inflation levels which hence lead to changes in inflation risk premium. Therefore investor can take out and in money from both bonds until we reach the equilibrium relationship in the two markets as mentioned in above equation (1).

Moreover, the expected return on conventional bonds depends on a liquidity risk premium. In fact, the yield of an inflation indexed bond (OAT) usually depend on liquidity risk premium in order to compensate investors for the risk of having to pay more than in the case of conventional bonds (OAT) to liquidate the OAT before its maturity.

Thus, we notice that this relationship is not yet theoretically presented nor empirically studied on French bond markets.

However, such relationship was extensively examined on both stock and bond markets. In this field, Wainscott (1990) using returns of US stocks and long term government bonds argued in favor of instability of the correlation between this two asset classes.

Besides, Fama and French (1996) as well as others, reject the hypothesis that stock and bond follow a pure random walk process. Whereas, Campbell and Taksler (2003) explore the impact of equity volatility on corporate bond yields.

So, to examine the above relation we use in this paper a Johansen cointegration test.

### 2.2 Cointegration

Cointegration, as defined and developed by Granger (1981) and Engle and Granger (1987), is a property of some nonstationary time series. If two or more nonstationary time series are cointegrated, a linear combination relationship being stationary is said to exist.

Cointegration can be also defined as a mathematical formulation of the long run relation between economic variables, especially the presence of the stationary long-run relations even if the variables themselves are nonstationary.

Various text books contain the basic aspects of cointegration, we can notice this in the studies of Hamilton (1994), Hansen and Johansen (1998) and Greene (2006).

In this prospect, many co-integration tests are developed. Tests of cointegration were developed for I(1) series by Shin (1994), based on the stationary test in Kwiatkowski, Phillips, Schmidt and Shin (1992), as well as by Saikkonen and Luukkonen (1993), Xiao and Phillips (2002), and others.

Before the application on the mentioned returns series, we briefly introduce the Johansen’s methodology co-integration.

The VAR model of order \( p \) is defined by:

\[
\Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + Bx_t + \varepsilon_t \quad (2)
\]

Where \( y_t \) is a \( k \)-vector of non-stationary \( I(1) \) variables, \( x_t \) is a \( d \)-vector of deterministic variables and \( \varepsilon_t \) is a vector of innovations. Matrices \( \Pi \) and \( \Gamma \) can be expressed as
\[ I = \sum_{i=1}^{p} A_j - I \quad \text{and} \quad \Gamma_i = -\sum_{j=i+1}^{p} A_j \]  

(3)

It’s noticed that Johansen’s method is based on the estimation of the \( I \) matrix from an unrestricted VAR and testing whether we can reject the restrictions implied by the reduced rank of \( I \). The trace statistics and Max-Eigen Statistic are usually used to make a decision about the acceptance or rejection of the co-integration. We recall that the trace statistic for the null hypothesis of \( r \) cointegrating relations is computed as \( \hat{\lambda}_{\text{trace}} = -T \sum_{i=r+1}^{k} \log(1-\hat{\lambda}_i) \), where \( \hat{\lambda}_{r+1},...,\hat{\lambda}_n \) are the smallest characteristic roots, and that the maximum eigen value statistic is \( \hat{\lambda}_{\text{max}} = -T \ln(1-\hat{\lambda}_{r+1}) \).

2.3 Subprime Crisis and Structural Break

The subprime crisis was extensively explained by economists, bankers, politicians and others, to learn about the causes, effects and consequences one can see Blackburn (2008), Gerardi et al. (2008). In this paper we emphasize on the effects of subprime on the relationship among OAT and OATi.

To achieve this aim, the break structure is analyzed by testing a model based on hypotheses below:

**H**\(_1\): Presence of a long run relation between OAT and OAT\(_i\) returns.

**H**\(_2\): The parameter \( \beta_i \) must be strictly positive: when OAT\(_i\) returns increase the OAT returns increase too.

**H**\(_3\): Temporal stability is verified if parameter \( \lambda \) is significantly equal to zero.

Assuming that the above hypotheses are true, the regression can be as follow:

\[ \ln(OAT_i) = \beta_0 + \beta_i \ln(OAT_i) + \lambda D_t + \epsilon_t \]  

(4)

With \( D_t = \begin{cases} 1 & \text{if } t \text{ is before subprime crisis} \\ 0 & \text{if not} \end{cases} \).

3. Data and Summary Statistics

3.1 Data

In an effort to examine the existence of relationship among conventional and index linked bonds, we use daily returns for French OAT\(_i\) with maturity of 2029 and OAT with maturity of 2029. The third variable used in this empirical study was the breakeven inflation rate.

Data are retrieved from “Agence France Trésor” web site. It concerns 2350 days of return observations (470 financial weeks) between January 2003 and March 2012.

The choice of this particular period is of importance to the fact that it includes the subprime crisis period.

The sample data was divided into three subsets, the first one before subprime crisis (2003 to June 2007), the second during the subprime crisis period (July 2007 to March 2008) and the last one after subprime crisis (April 2008 to 2012).

The definition of these different periods was based on Blackburn’s (2008) decomposition.
### 3.2 Summary Statistics

One of the usual assumptions in the theoretical finance literature is that the returns (logarithm of return) are normally distributed random variables. Let’s analyze tables hereafter to verify the theoretical assumption.

**Table 1** Descriptive statistics of OATi, OAT returns and breakeven before, during and after Subprime crisis

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Median</th>
<th>Std-dev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Jarque-Bera</th>
<th>Prob*</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Pre-crisis</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OATi</td>
<td>0.0209</td>
<td>0.0202</td>
<td>0.0043</td>
<td>0.0965</td>
<td>1.7992</td>
<td>72.293</td>
<td>0.000</td>
</tr>
<tr>
<td>OAT</td>
<td>0.0435</td>
<td>0.0431</td>
<td>0.0045</td>
<td>-0.121</td>
<td>1.6752</td>
<td>88.653</td>
<td>0.000</td>
</tr>
<tr>
<td>BE</td>
<td>0.0226</td>
<td>0.0225</td>
<td>0.0013</td>
<td>-0.030</td>
<td>2.950</td>
<td>0.2981</td>
<td>0.8614</td>
</tr>
<tr>
<td><strong>During crisis</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OATi</td>
<td>0.0228</td>
<td>0.0227</td>
<td>0.00888</td>
<td>0.2682</td>
<td>2.5012</td>
<td>4.3823</td>
<td>0.1117</td>
</tr>
<tr>
<td>OAT</td>
<td>0.0458</td>
<td>0.0456</td>
<td>0.00838</td>
<td>0.6772</td>
<td>3.178</td>
<td>15.241</td>
<td>0.0004</td>
</tr>
<tr>
<td>BE</td>
<td>0.0230</td>
<td>0.023</td>
<td>0.00457</td>
<td>-0.090</td>
<td>1.963</td>
<td>9.040</td>
<td>0.010</td>
</tr>
<tr>
<td><strong>Post crisis</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OATi</td>
<td>0.0175</td>
<td>0.0168</td>
<td>0.0042</td>
<td>0.2473</td>
<td>2.1782</td>
<td>39.213</td>
<td>0.000</td>
</tr>
<tr>
<td>OAT</td>
<td>0.0403</td>
<td>0.0402</td>
<td>0.0046</td>
<td>0.0703</td>
<td>2.5434</td>
<td>9.7295</td>
<td>0.0077</td>
</tr>
<tr>
<td>BE</td>
<td>0.0227</td>
<td>0.0232</td>
<td>0.0020</td>
<td>-0.990</td>
<td>4.236</td>
<td>232.33</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

* Probability associated to Jarque-Bera normality test.

One of the features which stand out most prominently from the first table is that the skewness of breakeven series is negative in the three periods, and then the distribution has a long left tail. The skewness of the OAT and the OATi is positive and very close to zero which implies that the distributions are symmetric or that they have a long right tail.

Except for the OAT during the subprime crisis and the breakeven after subprime crisis the kurtosis of all series is less than three and then, the distribution of both bonds and Break even are flat (platykurtic) relative to the normal.

Moreover, except for the breakeven series before subprime and the OATi during crisis, we reject the hypothesis of normal distribution at the 5% and 1% significance level.

### 4. Empirical Results

The three series representations in figure 1 show that the OATi returns and the break even move together over time. So, the break even can be regarded as a demand level for the OATi. Indeed, when demand for OATi increases the return of index linked bond increases too. We can also observe that OAT and OATi returns move together over time.

#### 4.1 Correlations

Table 2 on the next page presents the result of correlation between conventional bond, index linked bond and break even in different periods of time.

The correlation coefficient for all data set is about 0.938, very close to one, implying a strong positive correlation between the OAT return and OATi returns. The correlation between the two assets before subprime crisis is higher than that during crisis. This variation in correlation after and
before crisis can be explained as a change of structure in the relation between two assets as described by equation (1). As we can see from the findings, there is a clear pattern in correlation structure of these two bonds.

![Figure 1](image-url)  
**Figure 1** Daily returns of OAT and OATᵢ and daily break even before, during and after the subprime crisis

Obviously, such a correlation results are not sufficient. A long run relation between OAT and OATᵢ returns has to be studied. To insist on this aim, we apply a cointegration test.

### 4.2 Unit Root Test

Tests of the unit root (nonstationarity) hypothesis were developed by Fuller (1976), Dickey and Fuller (1979, 1981), Phillips and Perron (1988), and others.

To test the stationary of OAT and OATᵢ, we refer to an AFD and Philips and Peron tests.

Results of ADF and Phillips and Perron tests, applied to each of the time series, show that the null hypothesis of a unit root cannot be rejected. But a unit root is rejected for the first differences of the series. The series can thus be regarded as realizations of stochastic I(1) variables.

Such a result is confirmed for both series before and after the subprime crisis. It is important to note that ADF and Phillips and Perron test’s results show that the unit roots is rejected for the level of the two last series during the subprime crisis. Then, OAT and OATᵢ observed during the subprime crises are realizations of stochastic I(0) variables.

In contrast, ADF and Phillips and Perron tests show that the break even of inflation series are I(0) for all the data, pre and post crisis. But the last series can be regarded as realizations of stochastic I(1) during the subprime crisis. Since the presence of unit root in level OAT and OATᵢ series (for all the data, pre and post subprime crisis) it is important to examine co-integration relationship.

This paper uses the Johansen co-integration test to detect the number of co-integration relation.
Table 3 Stationary tests of AFD and Philips and Peron for OAT, OAT\textsubscript{i} and Breakeven

<table>
<thead>
<tr>
<th>Series</th>
<th>ADF level</th>
<th>PP level</th>
<th>ADF first diff</th>
<th>PP first diff</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OAT</td>
<td>-2.513</td>
<td>-2.2497</td>
<td>-44.864</td>
<td>-44.749</td>
</tr>
<tr>
<td></td>
<td>(0.112)</td>
<td>(0.188)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>OAT\textsubscript{i}</td>
<td>-2.829</td>
<td>-2.530</td>
<td>-45.458</td>
<td>-45.615</td>
</tr>
<tr>
<td></td>
<td>(0.0543)</td>
<td>(0.1083)</td>
<td>(0.0001)</td>
<td>(0.0001)</td>
</tr>
<tr>
<td>Breakeven</td>
<td>-4.175</td>
<td>-3.764</td>
<td>-35.364</td>
<td>-35.382</td>
</tr>
<tr>
<td></td>
<td>(0.0007)</td>
<td>(0.0034)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>OATB pre-crisis</td>
<td>-1.322</td>
<td>-1.329</td>
<td>-34.396</td>
<td>-34.394</td>
</tr>
<tr>
<td></td>
<td>(0.6207)</td>
<td>(0.6175)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>OAT\textsubscript{i} pre-crisis</td>
<td>-1.787</td>
<td>-1.775</td>
<td>-33.366</td>
<td>-33.363</td>
</tr>
<tr>
<td></td>
<td>(0.3871)</td>
<td>(0.3931)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Breakeven pre-crisis</td>
<td>-2.554</td>
<td>-2.537</td>
<td>-35.364</td>
<td>-35.382</td>
</tr>
<tr>
<td></td>
<td>(0.1029)</td>
<td>(0.1069)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td></td>
<td>(0.0129)</td>
<td>(0.0094)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>OAT\textsubscript{i} during crisis</td>
<td>-3.013887</td>
<td>-2.969539</td>
<td>-12.120</td>
<td>-12.120</td>
</tr>
<tr>
<td></td>
<td>(0.0353)</td>
<td>(0.0396)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Breakeven during crisis</td>
<td>-1.86572</td>
<td>-2.021977</td>
<td>-12.120</td>
<td>-12.120</td>
</tr>
<tr>
<td></td>
<td>(0.348)</td>
<td>(0.277)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>OAT post-crisis</td>
<td>-2.084093</td>
<td>-1.838429</td>
<td>-27.94477</td>
<td>-27.69772</td>
</tr>
<tr>
<td></td>
<td>(0.2513)</td>
<td>(0.3619)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>OAT\textsubscript{i} post-crisis</td>
<td>-2.205534</td>
<td>-1.938988</td>
<td>-29.05885</td>
<td>-29.06634</td>
</tr>
<tr>
<td></td>
<td>(0.2045)</td>
<td>(0.3144)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Breakeven post-crisis</td>
<td>-3.120625</td>
<td>-2.941213</td>
<td>-27.94477</td>
<td>-27.69772</td>
</tr>
<tr>
<td></td>
<td>(0.0254)</td>
<td>(0.0411)</td>
<td>(0.0000)</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>

Table 4 The Johansen co-integration tests among OAT and OAT\textsubscript{i} for all data

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Eigenvalue</th>
<th>Trace Statistic</th>
<th>5 Percent Critical Value</th>
<th>1 Percent Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>None **</td>
<td>0.007885</td>
<td>25.11463</td>
<td>15.41</td>
<td>20.04</td>
</tr>
<tr>
<td>At most 1 **</td>
<td>0.002776</td>
<td>6.527447</td>
<td>3.76</td>
<td>6.65</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Hypothesized No. of CE(s)</th>
<th>Max-Eigen Statistic</th>
<th>5 Percent Critical Value</th>
<th>1 Percent Critical Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>None ***</td>
<td>0.007885</td>
<td>18.58718</td>
<td>14.07</td>
</tr>
<tr>
<td>At most 1 **</td>
<td>0.002776</td>
<td>6.527447</td>
<td>3.76</td>
</tr>
</tbody>
</table>

Asterisks ** and *** imply rejection of the hypothesis at 5% and 1% significance level, respectively.

As shown in table 4, both trace and Eigen value statistics highlight 2 co-integrating equations at 5% level, but only one co-integrating equation at 1% level. So, there exists a long run relationship between the OAT and OAT\textsubscript{i} returns.

The Johansen co-integration test is applied for the two variables for the pre subprime crisis and post subprime crisis, and of course the results confirms the presence of a co-integration among variables at 5% level. This empirical result is consistent with the theoretical relationship between OAT and OAT\textsubscript{i} as indicated by equation (1). Thus, we can define the risk premium as the difference between the conditional expected OAT return and expected OAT\textsubscript{i} return.
4.3 Effects of Subprime Crisis and Structural Break

Co-integration test highlights a long run relationship between bonds. However, the subprime crisis during the period of about two years can disrupt this relationship. In order to see if the subprime crisis might change the relation we propose to estimate the econometric equation (4).

Table 5 Test of structural break and the effect of subprime crisis on the OAT_OATi returns relationship

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Standard Error</th>
<th>t-value</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ln(OATi)</td>
<td>1.9506</td>
<td>0.0142101</td>
<td>137.27</td>
</tr>
<tr>
<td>D</td>
<td>0.0299</td>
<td>0.0032</td>
<td>9.11</td>
</tr>
<tr>
<td>Constant</td>
<td>2.20</td>
<td>0.0455</td>
<td>48.41</td>
</tr>
</tbody>
</table>

Table 5 shows that there exist a positive relationship between the logarithm of returns at the level of OAT and OATi. We must recall that this model highlight only a short term relation between the two bonds. Notice that the long run relationship is verified using a Johansen test in the above paragraph.

Moreover, this model clearly shows the presence of structural change in the relation between the two types of bonds. Since, the parameter related to the dummy variable is significant.

This structural break in the relation between the two bonds returns is due to the increase of inflation and liquidity risk premium during the subprime crisis, corresponding to a period of large uncertainty.

For better understanding the presence of structural break, we analyze the volatility structure of OAT, OATi returns and the break even of inflation.

![Daily log returns of OAT](image1)

![Daily log returns of OATi](image2)

![Daily break even in 1st difference](image3)

Figure 2 Daily logarithmic returns of OAT and OATi and daily break even in first difference

The three figures show a presence of significant Volatility clustering, as noted by Mandelbrot³, “large changes tend to be followed by large changes, of either sign, and small changes tend to be followed by small changes”. Switching between high and low volatility states is probably the mechanism leading to volatility clustering and might explains the structural break in the OAT and OATi series due to the subprime crisis. We emphasize the presence of high volatility cluster one year after the subprime crisis.

It is also interesting to notice that there is a higher volatility of OAT returns than those of OATi. This difference in volatility can be explained by the fact that the OATi return’s volatility is

reduced by the effect of indexation to inflation rates. The major part of OAT\textsubscript{i} return’s volatility is due to the increase of liquidity risk during the crisis period.

The high volatility can be due to different reasons.

On one hand, the persistence of high volatility can be explained by the effect of expected inflation. Indeed, investors’ decisions depend on their expectation of inflation. So, during the subprime crisis period there is an uncertainty about the inflation level. As a consequence of this uncertainty, the bonds’ prices vary quickly inducing high volatilities.

On the other hand, the volatility cluster can be due to the liquidity risk. In fact, the yield of an inflation index linked bond consists of two components: a real yield and a liquidity risk premium to compensate investors for the risk of having to pay more to liquidate the OAT\textsubscript{i} before its maturity. So, during the subprime crisis the liquidity risk premium tend to increase because investors have to adjust their portfolios, making investors’ acceptance for liquidity risk decline. So, this phenomenon is commonly known as a “flight to quality” and avoids less risk assets.

After the subprime crisis, the liquidity is improved and thus the liquidity risk premium has decline. As a consequence, the volatility of OAT\textsubscript{i} returns decreased.

The volatility clustering can be also explained by examining the Breakeven of inflation rate behavior. Before we move, let’s recall that the breakeven inflation rate is yield spread between OAT and OAT\textsubscript{i} yields. So, we conclude an increase in the breakeven rate which can be viewed as a sign that market inflation expectations may be on the rise. Generally speaking, when the break even of inflation increases investors expect a high inflation, as a consequence an increase in volatility of index linked bond’s return.

Figure 2.b and 2.c show a volatility cluster in subprime crisis and confirms a high volatility of inflation breakeven rate which confirms the last idea.

5. Conclusion

This paper partially analyzes the relationship between OAT and OAT\textsubscript{i} returns. And, two main results are obtained:

First, the cointegration test indicates the existence of a long run relationship between the OAT and OAT\textsubscript{i} returns, and then the co-movement of OAT and OAT\textsubscript{i} is confirmed, a result which is believed to be important to understand the bond market’s mechanism. Second, the structural break test shows the presence of structural change in the relation between two types of bonds. This structural break is due to an increase of volatility in the OAT and OAT\textsubscript{i} returns in the subprime crisis period. Also, there is a higher volatility of OAT returns than those of OAT\textsubscript{i}. This difference in volatility can be explained by the fact that the OAT\textsubscript{i} return’s volatility is reduced by the effect of indexation to inflation rates.

To sum up, in any future work it would be desirable to extend this analysis in three directions:

First, it would be useful to know the mechanism of arbitration between two assets in the subprime crisis period.

Second, it would be interesting to investigate the presence of compensation between inflation risk premium and the liquidity risk premium.

Finally, it seems worthwhile to apply a time series model, such as GARCH, to analyze the volatility linkage in bond markets.
References