Marshall-Lerner Condition and the Balance of Payments
Constrained Growth: The Spanish Case

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Abstract: This paper develops an analytical reformulation of the Marshall-Lerner condition. This reformulation of the Marshall-Lerner condition leads us to reformulate both the Thirwall’s model and the Krugman’s rule in open economies, under the following terms: in the long run, if the Marshall-Lerner condition is maintained, the balance of payments constrained growth income not only depends on export and import income elasticities, but also on the cross-elasticity values between exports and imports. The analytical development has a series of implications for empirical studies about the Foreign Sector: export and import functions must be estimated considering the possibility of high cross correlations between them in the modelled countries. In the case of Spain, the long-term estimations of the price elasticities of exports and imports, and the respective cross elasticities, lead us to conclude that currency devaluation would, in the long term, improve the balance of trade and increase the constrained growth income in relation with the strong version of Thirwall’s law.

Keywords: Marshall-Lerner condition; Export and import flow simultaneity; Price-elasticity; Cross-elasticity

JEL Classification: F41, F44, F30, F11

1. Introduction

Traditionally, economic theory has analysed the foreign sector in relation to compliance with the Marshall-Lerner condition (Lerner, 1934; 1952): “a currency devaluation would have a positive impact on the trade balance when the sum of price elasticity of exports and imports is greater than 1”. This condition implicitly assumes that the GDP is independent from the exchange rate. In a globalised economy in which trade has heavily increased, there are many countries in which the ratio between the trade balance and the GDP is very high, so this assumption is not sustained. The theory of flexible exchange rates was developed, and it was shown that if the real exchange rate is flexible and the so-called Marshall-Lerner condition is satisfied, the balance of payments will equilibrate, without income adjustment. This may not be the case in the short run, or because of the nature of goods exported and imported by a particular country.

The economic literature includes numerous theoretical and empirical studies of the impact of exchange rate variations on the balance of trade; despite their number, they fail to agree on the effect of currency devaluation on trade balance, so it is an open question.
Thirwall (1982) developed a model started from proposition that no country can grow faster than that rate consistent with balance of payments equilibrium on current account, unless it can finance ever-growing deficits, which in general cannot. It is also the basis of Krugman’s rule (1989) that one country’s growth rate relative to another’s will be equiproportional to the ratio of its income elasticities of demands for exports and imports if the real exchange rate is constant. Oskooee-Bahmani (1998) employs a long-run method, cointegration technique, to estimate trade elasticities in less developed countries. In most cases the results reveal that indeed trade elasticities are large enough to support devaluation as a successful policy for improving the balance of trade.

Wilson (2001) analyses the impact of currency devaluation on the trade balances of Malaysia, Korea and Singapore, concluding that there is not J-curve for these countries, where the Marshall-Lerner condition is not met. Mahmud, Ullah and Yucel (2004), using non-parametric techniques to estimate the price elasticity of the exports and imports of six developed countries, find that the Marshall-Lerner condition is only partly met in some sub-sample periods. Mancies (2005), based on recent estimations of the Australian balance of trade, finds that the Marshall-Lerner condition is met in the 1999-2001 period. Pierdzioch (2005), using a general equilibrium model, reaches the conclusion that international capital mobility only increases the short-term effects on output if the Marshall-Lerner condition is met.

Matesanz and Fumorolas (2009), using multivariate cointegration tests and error-correcting models to obtain the determinants of the Argentinean balance of payments, find no empirical support for maintaining the Marshall-Lerner condition or existence of the J-curve in the short term. Hsing (2010) tests for the Marshall-Lerner condition in eight selected Asian countries and policy implications. The results show that the condition holds for India, Korea, Japan and Pakistan and can be rejected for Singapore and Malaysian.

Welfens (2012) studies the impact of foreign direct investment on the trade balance and concludes that: "for there to be a positive impact, higher elasticities are required than those maintained under the standard condition of Marshall and Lerner. Bahmani, Harvey and Hegerty (2013) show that although many studies suggest that the M-L condition is maintained, according to theirs researches it is not fulfilled in half of the cases. Sastre (2016) argues that in very open economies the independence between the exchange rate and the GDP is not maintained, so the M-L condition does not necessary have to be met. Bayar G. (2018) makes an exhaustive survey of the literature about the estimation of export equations and the econometrics techniques used.

2. The Theoretical Model

The economic determinants of the flows of exports and imports in small countries with very open economies, can be formulated through models of trade two countries with a representative agent (see Ostry, 1988; Obstfeld and Rogoff, 1995; Reinhart, 1995; Lombardo, 2001). The export and import demand functions are obtained by a dynamic optimisation process, in which the agent maximises his inter-temporal utility for the consumption of two types of goods: one produced on
The export and import demand for small, open economies would be, respectively:

\[ x = \varphi(G^f, m, tcr) \]  
\[ m = \varphi(G, x, tcr) \]

where \( \frac{G^f}{\partial tcr} = 0, \frac{\partial x}{\partial tcr} \neq 0, \frac{\partial x}{\partial m} \neq 0 \)

\[ \frac{\partial m}{\partial tcr} = 0, \frac{\partial m}{\partial x} \neq 0, \frac{\partial m}{\partial x} \neq 0 \]

G is the quantity of goods produced in the country (non-marketable); \( G^f \) is the quantity of non-marketable goods produced abroad and \( tcr \) is the real effective exchange rate.

These equations express the simultaneity between the flows of exports and imports and also the independence between the GDP and the exchange rate, and are the basis for reformulating the M-L condition and classifying the countries according to the cross-elasticities between their exports and imports.

We use the above export and import equations to analyze the Marshall-Lerner condition, starting with the general case of export and import simultaneity and proceeding to specific cases, including total independence between export and import flows, in which the Marshall-Lerner condition is maintained.

The balance of trade (BC) would be:

\[ BC = x - m = \varphi(Y^f, tcr, m) - tcr \cdot \varphi(Y, tcr, x) \]

Calculating the total impact of an exchange rate variation on the trade balance, we would have

\[ \frac{dB}{dtcr} = \frac{dx}{dtcr} - \frac{dm}{dtcr} \]

where \( \frac{dx}{dtcr} - \frac{dm}{dtcr} = \frac{dx}{dtcr} \cdot \frac{dm}{dtcr} - \frac{dx}{dm} \cdot \frac{dm}{tcr} + \frac{dm}{dcr} - [m - tcr \left( \frac{dm}{dx} \cdot \frac{dx}{dtcr} + \frac{dm}{dtcr} \right)] \]

Considering the price elasticities of exports and imports and their cross elasticities:

\[ \varepsilon_{x,tcr} = \frac{dx}{dtcr} \cdot \frac{tcr}{x}, \text{ or equivalently } \frac{dx}{dtcr} = \varepsilon_{x,tcr} \cdot \frac{x}{tcr} \]
\[ \varepsilon_{m,tcr} = \frac{dm}{dtcr} \cdot \frac{tcr}{m}, \text{ or equivalently } \frac{dm}{dtcr} = \varepsilon_{m,tcr} \cdot \frac{m}{tcr} \]
\[ \varepsilon_{m,x} = \frac{dm}{dx} \cdot \frac{x}{m}, \text{ or equivalently } \frac{dm}{dx} = \varepsilon_{m,x} \cdot \frac{m}{x} \]

\[ ^1 \text{Krugman (1995), in order to focus on the effects of Newly Industrializing Economies (NIEs), assumes a model consisting of only two economies: one that is intended to represent the OECD, the other to represent the aggregate of NIEs and assuming that the OECD faces a rest-of-world offer curve m=f(x).} \]

\[ \sim 31 \sim \]
And that at equilibrium, BC = 0, so \( m = \frac{x}{tcr} \). Replacing these expressions in equation (3), we obtain

\[
\frac{dBC}{dtcr} = \epsilon_{x,m} \cdot \frac{x}{m} \cdot \epsilon_{m,tcr} \cdot \frac{m}{tcr} + \epsilon_{x,tcr} \cdot \frac{x}{tcr} - m + tcr \cdot \left[ \epsilon_{m,x} \cdot \frac{m}{x} \cdot \epsilon_{x,tcr} \cdot \frac{x}{tcr} + \epsilon_{m,tcr} \cdot \frac{m}{tcr} \right]
\]

i.e.

\[
m \left[ \epsilon_{x,tcr} (1 + \epsilon_{m,x}) + \epsilon_{m,tcr} (1 + \epsilon_{x,m}) - 1 \right] = 0
\]

And then:

\[
\frac{dBC}{dtcr} = m \left[ \epsilon_{x,tcr} (1 + \epsilon_{x,m}) + \epsilon_{m,tcr} (1 + \epsilon_{m,x}) - 1 \right] = 0
\]

According to the above expressions, the balance of trade would be improved by a currency devaluation when \( \frac{dBC}{dtcr} > 0 \), and therefore:

\[
\epsilon_{x,tcr} (1 + \epsilon_{x,m}) + \epsilon_{m,tcr} (1 + \epsilon_{m,x}) > 1
\]  \hspace{1cm} (4)

### 3. Cross Elasticities of Export-Import, and Balance of Payments Constrained Growth

The model is based on Krugman’s rule that one country’s growth rate relative to another’s will be equiproportional to the ratio of its income elasticities of demand for exports and imports if the real exchange rate is constant (see Krugman, 1989 and Thirwall, 1982).

The simplest condition for a balance of payments in equilibrium is through the export and import demand functions. It follows that the rule of a balanced current account in the long term is

\[
P_d \cdot X = P_f \cdot M
\]

where \( P_d \) and \( P_f \) are the prices of exports and imports in domestic currency.

The export function depends on the relative prices of exports, the level of foreign income and imports

\[
X = k \cdot Y^e_f \cdot \left( \frac{P_d}{P_f} \cdot tc \right)^{\epsilon_{x,tc}} \cdot M^{\epsilon_{x,m}}
\]

where \( \epsilon_f \) is the income world elasticity of the exports; \( \epsilon_{x,tc} \) is the elasticity price of the exports and \( \epsilon_{x,m} \) is the cross elasticity export-import.

\[
M = k_1 \cdot Y^e_d \cdot \left( \frac{P_f}{P_d} \cdot tc \right)^{\epsilon_{m,tc}} \cdot X^{\epsilon_{m,x}}
\]

where \( \epsilon_d \) is the income elasticity of the imports; \( \epsilon_{m,tc} \) is the elasticity price of the imports and \( \epsilon_{x,m} \) is the cross elasticity import-export.

Transforming export and import into growth rates, we have the following system:

\[\sim 32 \sim\]
where lower-case letters stand for the growth rates variables.

Substituting (6) and (7) in (5) the equilibrium of the current account would be:

\[ p_d + \varepsilon_{x,tcr} (P_d - P_f - tc) + \varepsilon_f \cdot y_f + \varepsilon_{x,m} \cdot m \]
\[ m = \varepsilon_{m,tcr} (P_f - P_d + tc) + \varepsilon_d \cdot y_d + \varepsilon_{m,x} \cdot x \]

The balance of payments equilibrium growth rate \((y_d')\) would be:

\[ y_d' = \frac{(1+\varepsilon_{x,tcr}+\varepsilon_{m,tcr}+\varepsilon_f \cdot y_f + m \cdot \varepsilon_{x,m} \cdot x \cdot \varepsilon_{m,x})}{\varepsilon_d} \]

This result modify the Thirwal’s model (see Thirwall, 2011)

Analyzing this formula, we can outline several conclusions about the dynamics of the equilibrium rate.

First is the effect of the different inflation between the local economy and abroad. The effect on the equilibrium growth rate depends on the sum of the price elasticities of exports and imports. If this amount is greater than 1, an increase in the domestic inflation in relation with the abroad inflation will decrease the equilibrium growth rate if the exchange rate is constant.

Second, the rate of growth will depend on the difference between exports and imports elasticities weighted by the level of exports and imports that make up the external sector of each economic system.

We can consider the following four propositions for the cross elasticities between exports and imports.

**Proposition 1**

If \(\varepsilon_{x,m} = 0\) and \(\varepsilon_{m,x} = 0\), it characterises an economy that depends little on other countries, with zero correlation between exports and imports. Then we would have \(dBCE/dtcr > 0\) when

\[ \left( \varepsilon_{x,tcr} + \varepsilon_{m,tcr} \right) > 1 \] (9)

In this case, the Marshall-Lerner condition is maintained and the balance of payments equilibrium growth rate \((y_d')\) would be:

\[ y_d' = \frac{(1+\varepsilon_{x,tcr}+\varepsilon_{m,tcr}+\varepsilon_f \cdot y_f + m \cdot \varepsilon_{x,m} \cdot x \cdot \varepsilon_{m,x})}{\varepsilon_d} \]

If relative prices in international trade, or real exchange rates, are constant, equation (9) reduces to

\[ y_d' = \frac{\varepsilon_f \cdot y_f}{\varepsilon_d} \]

This is the strong version of Thirwall’s law.
Proposition 2

If \( \varepsilon_{m,x} \neq 0 \) and \( \varepsilon_{x,m} = 0 \), it characterises an economy in which the demand for imports depends on exports, and exports do not depend on imports. In this case \( \frac{dBC}{dtcr} > 0 \) when

\[
\left( \varepsilon_{x,tc} + \varepsilon_{m,tc} \left( 1 + \varepsilon_{m,x} \right) \right) > 1
\]

(11)

This condition would correspond to economies in which many industries import raw materials or intermediate products and then export the final products. Krugman (1995) defines it as “slicing up the production process” and suggests that it is one of the leading causes of growth in world trade. For some countries with very open economies, he proposes import equations like \( m = \phi(x, z) \), where \( x \) represents exports and \( z \) represents other determinants.

The balance of payments equilibrium growth rate (\( y_d^* \)) would be:

\[
y_d^* = \frac{(1+\varepsilon_{x,tc}+\varepsilon_{m,tc})+\varepsilon_f y_f - x \cdot \varepsilon_{m,x}}{\varepsilon_d}
\]

(12)

If relative prices in international trade, or real exchange rates, are constant, equation (n) could be reduced to

\[
y_d^* = \frac{\varepsilon_f y_f - x \cdot \varepsilon_{m,x}}{\varepsilon_d}
\]

This is lower than Thirwall’s law.

Proposition 3

If \( \varepsilon_{x,m} = 0 \) and \( \varepsilon_{x,m} \neq 0 \), this would represent an economy in which exports depend on imports, and imports would not depend on exports. Then, \( \frac{dBC}{dtcr} > 0 \) when

\[
\left( \varepsilon_{x,tc} \left( 1 + \varepsilon_{x,m} \right) + \varepsilon_{m,tc} \right) > 1
\]

(13)

This would correspond to the economies of countries used by multinational corporations as logistic bases for their products. The theory also depends on “slicing up the production process”. Multinational corporations do not react to unexpected changes in the demand for their products in the countries in which they operate by varying their production, which would lead to a significant increase in production costs, but by re-allocating their international stocks.

This process could be contemplated by the national accounts as imports and exports in the same period. Castillo and Picazo (1995) propose an indicator to measure “coincident trade”, defined as when a company exports and imports the same type of product at the same time, concluding that this type of trade represented nearly 12 per cent of all foreign trade in Spain in 1988.

The balance of payments equilibrium growth rate (\( y_d^* \)) would be:

\[
y_d^* = \frac{(1+\varepsilon_{x,tc}+\varepsilon_{m,tc})+(p_d-p_f-tc)+\varepsilon_f y_f + x \cdot \varepsilon_{m,x}}{\varepsilon_d}
\]

(14)

Similarly, if relative prices in international trade, or real exchange rates, are constant, equation (n) reduces to \( y_d^* = \frac{\varepsilon_f y_f + m \cdot x}{\varepsilon_d} \).

This result is higher than Thirwall’s law.

\(~ 34 ~\)
Proposition 4

If $\varepsilon_{m,x} \neq 0$ and $\varepsilon_{x,m} \neq 0$, these would apply to an economy in which import demand does not depend on export demand, and vice versa. In this case $dBC/dtcr > 0$, when

$$
\left( \varepsilon_{x,tc} \left( 1 + \varepsilon_{x,m} \right) + \varepsilon_{m,tc} \left( 1 + \varepsilon_{m,x} \right) \right) > 1
$$

(15)

In these economies, the empirical problem of estimating export and import flow determinants should be considered from the perspective of their simultaneity (see Mauleón and Sastre, 1992; 1996).

Sastre (2005) estimates a cointegrated simultaneous two-equation model for the balance of trade in Spain, with high explanatory capacity for both export and import flows, as well as the balance of trade and its evolution in 1967-2002.

The balance of payments equilibrium growth rate ($y^*_d$) would be:

$$
y^*_d = \frac{(1 + \varepsilon_{x,tc} + \varepsilon_{m,tc})(p_d - p_f - tc) + \varepsilon_f y_f + [m + \varepsilon_{x,m} - x + \varepsilon_{m,x}]}{\varepsilon_d}
$$

(16)

If relative prices in international trade, or real exchange rates, are constant, equation (16) similarly reduces to

$$
y^*_d = \frac{\varepsilon_f y_f + [m + \varepsilon_{x,m} - x + \varepsilon_{m,x}]}{\varepsilon_d}
$$

(17)

Higher or lower than Thirwall’s law? It depends on the cross elasticities between exports and imports.

If $\frac{m}{x} = \frac{\varepsilon_{m,x}}{\varepsilon_{x,m}}$, then equation (17) reduces to

$$
y^*_d = \frac{\varepsilon_f y_f}{\varepsilon_d}
$$

We also obtain the strong version of Thirwall’s law.

4. Simultaneity between Export and Import Flows: 
the Spanish Case

In relation to the Spanish economy, to study the long-run equilibrium relation between volume of imports and its determinants in one relation and the volume of exports and its determinants in another relation, we assume that the import and export demand equations take the following forms.

$$
\ln x = f(\ln r, \ln i, \ln m)
$$

(18)

$$
\ln m = f(\ln i, \ln p, \ln x)
$$

(19)

where $m$ is the volume of imports of goods and services; $r$ is the national investment; $r$ is the GDP of the OECD countries; and finally $i$ and $pr$, are the export and import price competitiveness indicators, respectively. The symbol $\ln$ stands for natural logarithm.
To establish whether a long-run equilibrium relationship exists between the variables in equations (18) and (19) for Spain, we use the Maximum Likelihood cointegration procedure proposed by Johansen (1988).

We apply the Johansen and Juselius (1990) method to determine the number of cointegrating vectors. The results of the $\lambda$-max and the trace showed the null hypothesis of no cointegration ($r = 0$) among all variables that enter into the import and export demand equations can be rejected at the 5% level of significance by setting its estimated coefficient.

In order to interpret the estimated cointegrating vectors, we normalize them on one of the variables by setting its estimated coefficient equal to -1, so we obtain long-run trade elasticities. This practice enables us to read the elasticities directly from cointegrating vectors. Applying and their corresponding test of the trace and the maximum eigenvalue are those expressed in the following table Johanssen (1988) methodology to the relation (5) and (6) for the period analysed, and assuming that the vector has a VAR(2) structure, the cointegration vectors obtained and their corresponding test of the trace and maximum eigenvalue are those expressed in the following table.

**Table 1. Tests of the trace and maximum eigenvalue**

<table>
<thead>
<tr>
<th>Long Run Import Equation</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln m = 0.84\ln r - 0.35\ln pr + 0.51\ln x$</td>
<td>(19.5)</td>
<td>(-18.5)</td>
<td>(19.5)</td>
</tr>
<tr>
<td>Tests</td>
<td>Value</td>
<td>Osterwald-Lenum 95%</td>
<td></td>
</tr>
<tr>
<td>Test $\lambda$-max</td>
<td>22.8</td>
<td>20.9</td>
<td></td>
</tr>
<tr>
<td>Test trace</td>
<td>30.5</td>
<td>29.7</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Long Run Export Equation</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\ln x = 1.20\ln ir - 1.80\ln it + 0.64\ln m$</td>
<td>(15.3)</td>
<td>(-5.05)</td>
<td>(18.0)</td>
</tr>
<tr>
<td>Tests</td>
<td>Value</td>
<td>Osterwald-Lenum 95%</td>
<td></td>
</tr>
<tr>
<td>Test $\lambda$-max</td>
<td>22.8</td>
<td>21.4</td>
<td></td>
</tr>
<tr>
<td>Test trace</td>
<td>30.5</td>
<td>30.3</td>
<td></td>
</tr>
</tbody>
</table>

In the brackets under each coefficient is t-statistics for each variable’s exclusion from the cointegrating space. Our long-run approach supports the notion that devaluation could improve the Spanish balance of trade.

As the Spanish economy, since the country joined the euro area, has constantly been reducing its competitiveness, linked to a high trade deficit, and considering that it can no longer alter its exchange rate as an economic policy tool, foreign trade balance adjustments necessarily involve a policy based on internal price and salary adjustments.
5. Conclusion

This paper develops an analytical reformulation of the Marshall-Lerner condition. The reformation in this paper has a series of implications for empirical studies aimed at modelling the export or import flows of a given country, or testing compliance with the Marshall-Lerner condition: export and import functions must be estimated considering the possibility of high cross correlations between exports and imports in the modelled countries. To test for the existence of a positive impact in the long term of a currency devaluation on the balance of trade should be verified for each country.

The paper’s analytical development leads us to reformulate the Thirwall’s model in open economies, under the following terms: In the long run, the balance of payments constrained growth income of countries with open economies depends not only on export and import income elasticities but also on the cross elasticities values between exports and imports.

In the case of Spain, the long-term estimations of the price elasticities of exports and imports, and the respective cross elasticities, lead us to conclude that currency devaluation would, in the long term, improve the balance of trade and increase the constrained growth income in relation with the strong version of Thirwall’s law.

References


