Abstract: In this paper we examine the optimal tax structure for a developing economy with international inter-family transfers. Our analysis characterizes conditions to identify the main determinants of the design of both the commodity and income tax. We find that these determinants change depending on the signs and values of the direct and cross price elasticities. A benchmark result of the literature suggests that the aggregate level of consumption, the weighted tax burdens from consumption and the efficiency costs of commodity taxes are the main determinants of a commodity tax. Our analysis suggests additional determinants, such as the aggregate level of income, the socially weighted aggregate level of labor income and the weighted aggregate elasticity of the supply of labor and income. A similar conclusion is obtained for the determinants of the income tax. Our analysis also provides a rationale to limit the use of the commodity tax and to increase the size of the income tax on the tax structure of this economy.

Keywords: Equity, Altruism, Remittances, Efficiency, Optimal tax structure

JEL Classifications: D63, D64, F24, H21

1. Introduction

The government performs fundamental and core tasks in the economy, such as the provision of public goods, the guarantee of the rule of law, and the implementation of anti-poverty programs. The intervention of government requires the financing of the government’s programs which, in most modern economies, is supported by a tax structure that provides the government the financial means for its activities. Since the government is in need of implementing taxes, there is a large literature in public economics that asks the following question: what is the optimal tax structure that collects an adequate level of tax revenue? For literature reviews see Auerbach and Hines (2002) and Mirrlees (1986).

Another line of research that has received the attention in the literature of public economics is the economic impact of altruism. For this spirit of study, see Cox (1990), Andreoni (1988), Becker (1974), among many others. In this paper we consider that international private transfers can be interpreted as an expression of altruism. According to the report of the international migration outlook of the OECD (2006), international inter-family transfers are playing an important role in developing countries, with remittances having significant effects on private consumption, savings, the balance of payments, the distribution of income, among other economic outcomes.

On the issue that remittances affect the distribution of income, Acosta et al. (2007) find that remittances in Latin American and Caribbean (LAC) countries have reduced the inequality in the distribution of income. Adams and Page (2005) find that remittances significantly diminished the
level of poverty in developing countries. However, remittances might not only reduce inequality but also increase it. Stark et al. (1986, 1988) argue that the recipients of remittances can be predominantly high (low) income families and therefore remittances are likely to increase (reduce) the inequality in the distribution of income.

In this paper we are interested in studying the optimal tax structure for a developing economy with international inter-family transfers that are explained by the interdependence of utility of individuals (altruism). The analysis of optimal taxation for this kind of economy has not received adequate attention in the literature. There are a couple of novel elements that are relevant for tax policy design: first, international private transfers are a market mechanism that redistributes income among families and therefore remittances change the welfare calculus associated with the optimal distribution of tax burdens (see Kochi and Ponce 2011). Second, private transfers are also relevant for the analysis of the efficiency of the tax system. A donor family (a family who provides an international private transfer) decides the level of consumption, the supply of labor and the size of the private transfer. Taxes affect the relative prices of consumption, the donation of money to another family, the supply of labor of donors and the size of remittances. A benevolent social planner should recognize the behavioral effects of taxation on these economic decisions.

The objective of this paper is to provide an answer to the question: what is the optimal tax structure for an economy with international private transfers? The contribution of this paper to the literature is that our model produces new insights about the main determinants of the optimal level of the commodity tax and the labor income tax for a developing economy with international private transfers.

The rest of the paper is structured as follows. Section two contains a brief review of the literature. Section three considers the theoretical model with the main propositions of the paper. Section four includes comments on the implications of our analysis on tax policy, and section 5 concludes.

2. Literature Review

The literature of optimal taxation is at the core of public economics (for comprehensive literature reviews, see Auerbach and Hines (2002) and Mirrlees (1986)). Ramsey (1927) is the first to analyze the problem of a commodity tax structure that seeks to provide an amount of tax revenue for the government. The structure of the optimal taxation theory is as follows: households seek to maximize their wellbeing by deciding to consume goods and to supply labor subject to a budget constraint. The government is ruled by a benevolent social planner who seeks to implement a tax structure that maximizes a social welfare function taking into account how taxes affect the behavior of economic agents. Since taxes distort the relative prices of goods and services, the economic decisions of households are also distorted. A benevolent social planner recognizes that an optimal tax structure should minimize the excess burden of taxation.

A benevolent social planner also recognizes that taxes have a negative impact on the welfare of economic agents. Different forms of taxation will lead to different distributions of the welfare costs of taxes due to the distribution of preferences, earning abilities and labor income of households. Hence an optimal tax structure would seek to minimize the overall costs from taxation and, by equity considerations, would take into account the distribution of these costs in the design of taxes.

Moreover, different taxes appeal, more than others, to policy makers because of the relative ability of a particular tax rate to collect tax revenue. A marginal increase in tax revenue might be different from one tax rate to another because of the definition of the tax base (which depends on
the type of exclusions and exemptions of the tax base) and the excess burden from taxation. Hence a benevolent social planner also takes into account differences in marginal tax revenue from different taxes to constitute an optimal tax structure.


The literature on the role of altruism in the design of tax policy has focused on the rationale on government intervention (see Hochman and Rodgers (1969)), on how different types of altruistic behavior affect optimal Pigouvian taxes (see Johansson, 1997), on the implications for the form of public transfers vis-a-vis in-kind transfers (see Coate, 1995), on the existence of majority rule equilibria that involves progressive taxation (see Kranich, 2001), and on the effect of private transfers on the government’s redistributive policy under targeted and universal redistributive public spending (see Kochi and Ponce (2010, 2011)).

In this paper we develop a simple model to study the main determinants of the optimal design of a commodity and a linear income tax under two types of equilibriums: in the first equilibrium, we assume that cross price elasticities are zero, that is, the aggregate elasticity of labor with respect to the commodity tax and the aggregate elasticity of consumption with respect to the labor income tax are zero to establish the benchmark model. In the second equilibrium, the cross price elasticities are not zero. We develop a comparative analysis between these two types of equilibriums and identify conditions in which there are significant differences between the determinants of the optimal design of a commodity tax and a linear income tax.

In the first equilibrium, the main determinants of the commodity tax are the aggregate level of consumption, a socially weighted aggregate level of consumption and the weighted aggregate elasticity of consumption with respect to the commodity tax. Similarly, the optimal income tax rate depends on the aggregate level of income, a socially weighted aggregate level of labor income, and a weighted aggregate elasticity of the supply of labor and income taxes. These outcomes can be considered as the benchmark results in the literature (for similar results for economies that do not consider international private transfers, see Myles, 1995).

For the second equilibrium we find that the optimal commodity and income taxes are determined by a linear combination of the benchmark cases of the commodity tax and the income tax found in the first equilibrium. One interesting result is that there are values of the aggregate consumption-commodity tax elasticity, the elasticity of the supply of labor-commodity tax, the consumption-income tax and the labor supply-income tax elasticities, in which the main determinants of a commodity tax are the typical determinants of a labor income tax and vice versa. This result has not previously found in the literature but it has interesting implications for the relative merits of using commodity versus income taxation in a tax structure.
3. Tax Policy of the Government

In this economy there is a household who receives private transfers from another household. The preference of a household receiving inter-family transfers are given by a strict quasi-concave utility function \( \mu = \mu(c, 1 - \ell) \) where \( c \) is consumption, \( 1 - \ell \) is leisure and \( \ell \) is the supply of labor. The budget constraint of this family is \( (1 + t)c = w\ell(1 - \tau) + R \), where \( (1 + t) \) is the consumer’s price of the private good \( c \), and \( t \) is a proportional commodity tax, \( w\ell(1 - \tau) \) is the after-tax labor income, \( w \) is the labor wage, \( \tau \) is a proportional tax on labor income and \( R \) is the amount of private transfers received by this family. In this economy, markets are competitive and the wage income reflects the household’s earning ability. In this economy, there is heterogeneity in the households’ earning abilities, hence wages are given by the interval \( w \in [w_o, w_{\max}] \) and \( h(w) \in \mathbb{R}_+ \), where \( h(w) \) represents the number of families with wage \( w \).

The budget constraint of the household who receives private transfers reflects that families can have several sources of income and that at least one source of income might be difficult to tax under the income tax system.\(^1\) The indirect utility function of the household who receive private transfers and wage \( w \) is characterized by

\[
v(t, \tau, w, R) = \text{Max} \{ \mu(c^*, 1 - \ell^*) \} \text{ s.t. } (1 + t)c^* = w\ell^*(1 - \tau) + R \]

Following Becker (1974), we assume that families that provide inter-family transfers are altruistic and care about the utility of another family.\(^3\) The household who provides inter-family transfers (the donor family) lives abroad and sends remittances to a family living in the home country. The preferences of households sending remittances are represented by a strict quasi-concave function given by \( \mu^d = \mu^d \left( c^d, 1 - \ell^d, v(t, \tau, w, R) \right) \) where \( c^d \) is consumption, \( 1 - \ell^d \) is leisure, and \( \ell^d \in [0,1] \) is the supply of labor of the donor family (here the superscript \( d \) denotes the donor). Moreover, \( v(t, \tau, w, R) \) is the indirect utility of the family receiving private transfers, and we assume that \( \partial \mu^d / \partial v > 0 \).

The budget constraint for donors of private transfers is given by \( c^d = w^d\ell^d - R \), where \( w^d \) is the wage of the donor household. The distribution of wages of donor families is given by \( w^d \in [w^d_o, w^d_{\max}] \). Donor Households seek to maximize their utility by selecting their consumption, labor supply, and the size of private transfers that will be sent to another family.\(^4\)

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\(^1\) In our model, the tax authority seeks to collect tax revenue from labor income and private transfers are not taxed at the income tax rate. This condition captures the idea that remittances could not be easy to monitor and taxed effectively (due to enforcement costs) under an income tax. Also note that since remittances are excluded from the income tax then the commodity tax and the income tax are not equivalent because their tax bases are not equivalent.

\(^2\) Where \( c^*(t, \tau, w, R), \ell^*(t, \tau, w, R) \in \text{argMax} \mu(c, 1 - \ell) \) subject to \( (1 + t)c = w\ell(1 - \tau) + R \). In some sections of the paper we will use the notation \( c^*, \ell^* \) to represent \( c^*(t, \tau, w, R), \ell^*(t, \tau, w, R) \) to save space.

\(^3\) Becker (1984) considers the case in which private transfers are explained by altruism. However, Bernheim et al. (1985) provide a model in which private transfers are explained by economic exchange. Our choice of considering private transfers throughout altruism is explained by the fact that the phenomenon of altruism has received significant theoretical attention, see Cox and Jimenez (1989, 1990).

\(^4\) For simplicity of the analysis we assume that the government cannot tax families living abroad (that is, families sending remittances). This assumption has empirical support for developing countries in which the tax authority can face administrative (political) difficulties to tax income earned abroad.
We consider the problem of a benevolent social planner that selects a proportional commodity tax $t$ and a proportional labor income tax $\tau$ to maximize a weighted social welfare function subject to the constraint that the tax system yields a certain amount of tax revenue $\bar{TT}$. Formally, the problem of tax policy design of the government is

$$\text{Max } \Psi = \int_{w_0}^{w_{\text{max}}} \psi(w)h(w)v(t,\tau,w,R)dw$$

subject to

$$\bar{TT} = t \int_{w_0}^{w_{\text{max}}} h(w)c^*(t,\tau,w,R)dw + \tau \int_{w_0}^{w_{\text{max}}} h(w)w\ell^*(t,\tau,w,R)dw$$

where $\Psi$ is a social welfare function, $\psi(w) \in \mathbb{R}_+$ is the weight in the social welfare function of family type $w$. Equation (2) says that the objective of collecting a given amount of tax revenue, $\bar{TT}$, depends on both the commodity and labor income taxes on residents and the aggregate level of consumption and labor income of families living in the home country.

**Definition.** The economic equilibrium for this economy is characterized as follows:

a) The government selects taxes $t^*$ and $\tau^*$, such that

$$t^*,\tau^* \in \text{arg max } \Psi = \int_{w_0}^{w_{\text{max}}} \psi(w)h(w)v(t,\tau,w,R)dw$$

subject to

$$\bar{TT} = t \int_{w_0}^{w_{\text{max}}} h(w)c^*(t,\tau,w,R)dw + \tau \int_{w_0}^{w_{\text{max}}} h(w)w\ell^*(t,\tau,w,R)dw$$

b) Households with $w \in [w_0,w_{\text{max}}]$ who receive private transfers select their consumption and labor choices such that:

$$c^*(t,\tau,w,R),\ell^*(t,\tau,w,R) \in \text{arg max } \left\{ \mu(c,1-\ell) \left\{ \begin{array}{l} s.t. (1+t)c = w\ell(1-\tau) + R \end{array} \right\} \right\}$$

(c) Donors of private transfers with a wage $w^d \in [w_0^d,w_{\text{max}}^d]$ decide their labor supply $\ell^d(t,\tau,w^d,R)$, consumption $c^{d^*}(t,\tau,w^d,R)$, and the size of private transfers to a household type $w$, $R^*(t,\tau,w^d,w)$, to maximize the donor’s utility such that

$$c^{d^*},\ell^{d^*},R^* \in \text{arg max } \left\{ \mu^d(c^{d^*},\ell^{d^*},v(t,\tau,w,R)) \right\}$$

subject to

$$c^{d^*} = w^d\ell^{d^*} - R$$

Our definition of the economic equilibrium recognizes that the government decides the optimal tax structure $t^*,\tau^*$ by taking into account the relative abilities of taxes to collect tax revenue and the effects of taxation on the efficiency and equity in the allocation of resources. Efficiency considerations on tax policy come from the fact that taxes change the relative prices of consumption, the supply of labor and the decision to send private transfers, and also distort these

However, it is common that developed economies have more efficient tax authorities that can collect some tax revenue from families earning income abroad. We will leave this latter case for future research.

We follow the literature and assume $\bar{TT}$ is given.

To simplify the notation we will use $c^{d^*},\ell^{d^*},R^*$ as a shortcut of $c^{d^*}(t,\tau,w^d,w),\ell^{d^*}(t,\tau,w^d,w)$ and $R^*(t,\tau,w^d,w)$ when we consider convenient.
households’ choices. The distortion of prices by the government’s taxes also leads to deadweight costs of taxation. A benevolent social planner would seek to select the commodity and income taxes as to minimize the distortionary impact of taxation on the households’ economic decisions with respect to consumption and the supply of labor.

The distribution of tax burdens from the commodity and income taxes will also depend on the distribution of labor income and private transfers. A benevolent social planner will also consider the distribution of tax burdens for families. Hence, the equity in the allocation of resources plays a significant role in determining the optimal tax rates of commodity and income taxation.

Our definition of the economic equilibrium also characterizes the economic behavior of families who receive private transfers and decide their choices of consumption and their supply of labor by taking into account their income from labor services, the private transfers received, and the taxes imposed by the government. Finally, a rational donor type \( w^d \) decides consumption, their supply of labor and the size of remittances.

For the analysis that follows, it is convenient to define the aggregate weighted elasticity of the labor supply and the commodity tax \( t \) as \( \varepsilon^A_{\ell-t} \) and the aggregate weighted elasticity of consumption with respect to the income tax as \( \varepsilon^A_{c-t} \). These are given by:

\[
\varepsilon^A_{\ell-t} = \int_{w_0}^{w_{\text{max}}} h(w)w\ell^*\varepsilon^t_{\ell-t}|_{t=0} dw
\]

where \( \varepsilon^t_{\ell-t}|_{t=0} = \frac{\partial \ell^*(1+t)}{\partial t} \ell^* \) when \( t = 0 \), and

\[
\varepsilon^A_{c-t} = \int_{w_0}^{w_{\text{max}}} h(w)c^*\varepsilon^t_{c-t}|_{t=0} dw
\]

where \( \varepsilon^t_{c-t}|_{t=0} = \frac{\partial c^*(1-t)}{\partial t} c^* \) is evaluated at \( t = 0 \).

Similarly, the aggregate weighted elasticity of consumption with respect to the commodity tax, \( \varepsilon^A_{c-t} \), is given by:

\[
\varepsilon^A_{c-t} = \int_{w_0}^{w_{\text{max}}} h(w)c^*\varepsilon^t_{c-t}|_{t=0} dw
\]

With \( \varepsilon^t_{c-t}|_{t=0} = \frac{\partial c^*(1+t)}{\partial t} c^* \) when \( t = 0 \). Finally, the aggregate weighted elasticity of the supply of labor with respect to the labor income tax, \( \varepsilon^A_{\ell-t} \), is yielded as:

\[
\varepsilon^A_{\ell-t} = \int_{w_0}^{w_{\text{max}}} h(w)w\ell^*\varepsilon^t_{\ell-t}|_{t=0} dw
\]

where \( \varepsilon^t_{\ell-t}|_{t=0} = \frac{\partial \ell^*(1-t)}{\partial t} \ell^* \) is evaluated at \( t = 0 \).

**Proposition 1.** If \( \varepsilon^A_{\ell-t} = \varepsilon^A_{c-t} = 0 \), then the solution of the tax policy design problem for the government is given by \( \tilde{t}^* \) and \( \tilde{\ell}^* \) satisfying:

\[~ 18 ~\]
\[ \bar{\tau}^* = \left\{ -\frac{1}{\varepsilon_{c^{-1}}} \right\} \left\{ \int_{w_0}^{w_{\text{max}}} h(w)c^* dw - \frac{1}{\lambda^*} \int_{w_0}^{w_{\text{max}}} h(w)\psi(w)ac^* dw \right\} \] (11)

\[ \bar{\tau}^* = \left\{ -\frac{1}{\varepsilon_{\ell^{-1}}} \right\} \left\{ \int_{w_0}^{w_{\text{max}}} h(w)w\ell^* dw - \frac{1}{\lambda^*} \int_{w_0}^{w_{\text{max}}} h(w)\psi(w)aw\ell^* dw \right\} \] (12)

where \( \alpha \) is the household's marginal utility of income and \( \lambda^* \) is the social marginal gain from relaxing the government's constraint associated with the collection of tax revenue.

**Proof:** Define \( \lambda^* \) is the Lagrange multiplier associated with condition (2). The first-order necessary conditions for the two tax policy problems of the government are:

\[ \int_{w_0}^{w_{\text{max}}} \psi(w)\frac{\partial v}{\partial t} dw + \lambda^* \int_{w_0}^{w_{\text{max}}} h(w)c^* dw = 0 \quad \forall \bar{\tau}^* > 0 \] (13)

\[ \int_{w_0}^{w_{\text{max}}} \psi(w)\frac{\partial v}{\partial t} dw + \lambda^* \int_{w_0}^{w_{\text{max}}} h(w)w\ell^* dw = 0 \quad \forall \bar{\tau}^* > 0 \] (14)

\[ \bar{T} = \int_{w_0}^{w_{\text{max}}} h(w)c^* dw + \bar{\tau}^* \int_{w_0}^{w_{\text{max}}} h(w)w\ell^* dw \] (15)

Condition \( -\varepsilon_{c^{-1}} = \varepsilon_{c^{-1}} = 0 \) means that \( \frac{\partial c^*}{\partial t} = \frac{\partial c^*}{\partial t} = 0 \). By defining \( \alpha \) as the household’s marginal utility of income and using \( \frac{\partial v}{\partial t} = -ac^* \) into (13) and \( \frac{\partial v}{\partial t} = -aw\ell^* \) into (14), we can solve the system of equations and re-arrange terms to yield that

\[ \bar{\tau}^* = \left\{ -\frac{1}{\varepsilon_{c^{-1}}} \right\} \left\{ \int_{w_0}^{w_{\text{max}}} h(w)c^* dw - \frac{1}{\lambda^*} \int_{w_0}^{w_{\text{max}}} h(w)\psi(w)ac^* dw \right\} \] (16)

\[ \bar{\tau}^* = \left\{ -\frac{1}{\varepsilon_{\ell^{-1}}} \right\} \left\{ \int_{w_0}^{w_{\text{max}}} h(w)w\ell^* dw - \frac{1}{\lambda^*} \int_{w_0}^{w_{\text{max}}} h(w)\psi(w)aw\ell^* dw \right\} \] (17)

Proposition 1 implies that if the aggregate elasticity of labor with respect to the commodity tax \( t, \varepsilon_{c^{-1}} \), and the aggregate elasticity of consumption with respect to the labor income tax \( \tau, \varepsilon_{\ell^{-1}} \), are zero, then the optimal commodity tax \( \bar{\tau}^* \) depends positively on the difference between the aggregate level of consumption and the socially weighted aggregate level of consumption, \( \left\{ \int_{w_0}^{w_{\text{max}}} h(w)c^* dw - \frac{1}{\lambda^*} \int_{w_0}^{w_{\text{max}}} h(w)\psi(w)ac^* dw \right\} \). Equity considerations are determined by the expression of \( -\frac{1}{\lambda^*} \int_{w_0}^{w_{\text{max}}} h(w)\psi(w)ac^* dw \). This implies that the higher this term the higher are the social costs associated with commodity taxation and the lower should be \( \bar{\tau}^* \) at the equilibrium. The commodity tax \( \bar{\tau}^* \) also depends negatively on the inefficiency costs from taxation determined by the weighted aggregate elasticity of consumption with respect to \( t \). This means that the higher \( -\varepsilon_{c^{-1}} > 0 \), the higher the inefficiency costs of taxation and the lower is \( \bar{\tau}^* \) at the equilibrium.

A similar interpretation is given to the optimal rate of the labor income tax \( \bar{\tau}^* \) which is increasing with higher positive differences between the aggregate level of income and the socially weighted aggregate level of labor income, \( \left\{ \int_{w_0}^{w_{\text{max}}} h(w)w\ell^* dw - \frac{1}{\lambda^*} \int_{w_0}^{w_{\text{max}}} h(w)\psi(w)aw\ell^* dw \right\} \).
and $\tilde{\tau}^*$ is falling with increases in the weighted aggregate elasticity of the supply of labor and income taxes $\varepsilon_{\ell-t}^A$ due to efficiency considerations. This says that the higher $-\varepsilon_{\ell-t}^A > 0$, the higher the inefficiency costs of taxation and the lower is $\tilde{\tau}^*$ at the equilibrium. Equity considerations are explained by the expression of $-\frac{1}{\lambda^t} \int_{w_0}^{w_{\max}} h(w) \psi(w)aw\ell^* dw$. This means that the higher this term the higher are the social costs from labor income taxation and the lower should be $\tilde{\tau}^*$ at the equilibrium.

**Proposition 2.** Consider $\tilde{\tau}^*$ and $\tilde{\tau}^*$ are given by conditions (11) and (12) of proposition 1. If $\varepsilon_{\ell-t}^A \neq 0$ and $\varepsilon_{\ell-t}^A \neq 0$ then the solution of the tax policy design problem for the government is given by $t^*$ and $\tau^*$ satisfying

\[
t^* = \theta \tilde{\tau}^* + (1 - \theta) \left( \frac{\varepsilon_{\ell-t}^A}{\varepsilon_{\ell-\tau}^A} \right) \tilde{\tau}^*
\]

\[
\tau^* = \theta \tilde{\tau}^* + (1 - \theta) \left( \frac{\varepsilon_{\ell-t}^A}{\varepsilon_{\ell-\tau}^A} \right) \tilde{\tau}^*
\]

where

\[
\theta = \frac{\varepsilon_{\ell-t}^A}{\varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^\ell_{\ell-t}} \quad \text{and} \quad (1 - \theta) = \frac{-\varepsilon_{\ell-t}^A}{\varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^\ell_{\ell-t}}
\]

**Proof:** The first-order conditions for this problem are:

\[
\int_{w_0}^{w_{\max}} \psi(w)h(w) \frac{\partial v}{\partial t} dw + \lambda^t \int_{w_0}^{w_{\max}} h(w)c^* dw
\]

\[
+ \lambda^t t^* \int_{w_0}^{w_{\max}} h(w) \frac{\partial c^*}{\partial t} dw + \lambda^t \tau^* \int_{w_0}^{w_{\max}} h(w)w \frac{\partial \ell^*}{\partial t} dw = 0 \quad \forall \tilde{\tau}^* > 0
\]

\[
\int_{w_0}^{w_{\max}} \psi(w)h(w) \frac{\partial v}{\partial t} dw + \lambda^t \int_{w_0}^{w_{\max}} h(w)w \ell^* dw
\]

\[
+ \lambda^t t^* \int_{w_0}^{w_{\max}} h(w) \frac{\partial c^*}{\partial t} dw + \lambda^t \tau^* \int_{w_0}^{w_{\max}} h(w)w \frac{\partial \ell^*}{\partial t} dw = 0 \quad \forall \tilde{\tau}^* > 0
\]

\[
\bar{Tr} = t^* \int_{w_0}^{w_{\max}} h(w)c^* dw + \tau^* \int_{w_0}^{w_{\max}} h(w)w \ell^* dw
\]

As before, $\lambda^t$ is the Lagrange multiplier associated with condition (2) for the solution of the tax structure $t^*$ and $\tau^*$. To solve for the tax structure, we define the following system of equations:

\[
\begin{bmatrix}
-\varepsilon_{\ell-t}^A & -\varepsilon_{\ell-t}^A \\
-\varepsilon_{\ell-t}^A & -\varepsilon_{\ell-t}^A
\end{bmatrix}
\begin{bmatrix}
t^* \\
\tau^*
\end{bmatrix}
= 
\begin{bmatrix}
\int_{w_0}^{w_{\max}} h(w)c^* dw - \frac{1}{\lambda^t} \int_{w_0}^{w_{\max}} h(w)\psi(w)ac^* dw \\
\int_{w_0}^{w_{\max}} h(w)w\ell^* dw - \frac{1}{\lambda^t} \int_{w_0}^{w_{\max}} h(w)\psi(w)aw\ell^* dw
\end{bmatrix}
\]

Define the determinant of the system in (24) as $H = \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A$ and show that

\[
t^* = \left\{ \frac{1}{H} \right\} \left( \frac{\varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A}{\varepsilon_{\ell-\tau}^A} \right) \tilde{\tau}^* - \varepsilon_{\ell-t}^A \tilde{\tau}^* - \varepsilon_{\ell-t}^A \tilde{\tau}^* 
\]

\[
\sim 20
\]
where

\[
\tilde{\tau}^* = \left\{ \frac{1}{H} \right\} \left\{ \varepsilon_{\tilde{c}-\tilde{t}} \tilde{\tau}^* - \varepsilon_{\tilde{c}-\tilde{t}} \tilde{\tau}^* \right\}
\]  

(26)

Define \( \theta \) as follows:

\[
\theta = \frac{\varepsilon_{\tilde{c}-\tilde{t}} \varepsilon_{\tilde{c}-\tilde{t}}}{H}
\]  

(29)

which implies \((1 - \theta) = \frac{-\varepsilon_{\tilde{c}-\tilde{t}} \varepsilon_{\tilde{c}-\tilde{t}}}{H}\). Use (29) into (27) and rearrange terms to show that conditions (24) and (25) can be expressed as follows, respectively:

\[
t^* = \theta \tilde{t}^* + (1 - \theta) \left\{ \frac{\varepsilon_{\tilde{c}-\tilde{t}}}{\varepsilon_{\tilde{c}-\tilde{t}}} \right\} \tilde{\tau}^* 
\]  

(30)

\[
\tau^* = \theta \tilde{\tau}^* + (1 - \theta) \left\{ \frac{\varepsilon_{\tilde{c}-\tilde{t}}}{\varepsilon_{\tilde{c}-\tilde{t}}} \right\} \tilde{\tau}^* 
\]  

(31) #

Proposition 2 says that if \( \varepsilon_{\tilde{c}-\tilde{t}} \neq 0 \) and \( \varepsilon_{\tilde{c}-\tilde{t}} \neq 0 \), then the optimal commodity tax \( t^* \) is a linear combination of the commodity tax \( \tilde{t}^* \) (given by condition (11) in Proposition 1) and the labor income tax \( \tilde{t}^* \) (given by condition (12) in Proposition 1). The commodity tax \( \tilde{t}^* \) has a weight of \( \theta \) in determining the optimal tax rate \( t^* \). This weight depends on a combination of the aggregate price elasticity of consumption-commodity tax, the cross price elasticity of the supply of labor-commodity tax, the consumption-income tax elasticity and the labor supply-income tax elasticity (see condition (20)). The commodity tax \( \tilde{t}^* \) also depends on the labor income tax \( \tilde{t}^* \) with a weight \( (1 - \theta) \). A similar interpretation is given to \( \tau^* \) which is linear combination of the labor income tax \( \tilde{\tau}^* \) with a weight \( \theta \) and the commodity tax \( \tilde{\tau}^* \) with a relative weight of \( (1 - \theta) \).

This outcome has the following interesting implications: first, a commodity tax \( t^* \) not only depends on aggregate consumption, the socially weighted aggregate consumption and the efficiency costs from commodity taxation on consumption (as it is commonly considered and characterized by condition (11)), but also \( t^* \) depends on \( \tilde{t}^* \) which means that \( t^* \) depends on aggregate labor income, a socially weighted aggregate labor income and the efficiency costs from taxes on labor earnings. Similarly, a labor income tax \( \tau^* \) not only depends on aggregate labor income, the socially weighted aggregate labor income and the efficiency costs from the labor income tax (as it is commonly considered and characterized by condition (12)), but also it depends on aggregate consumption, the socially weighted aggregate consumption and the efficiency costs from commodity taxation on consumption because \( \tau^* \) is a function of \( \tilde{\tau}^* \).

These outcomes mean that the optimal commodity tax, \( t^* \), should recognize the distortionary effects of the income tax (which tends to reduce the optimal level of \( \tilde{\tau}^* \)) and the optimal level of the income tax, \( \tau^* \), should recognize the efficiency costs of commodity taxation (which also tends to reduce the equilibrium level of \( \tilde{\tau}^* \)). Similarly, the optimal commodity tax, \( t^* \), should recognize, by
equity considerations, the distribution of welfare costs of the income tax (which tends to reduce the optimal level of \( \bar{t} \)) and the optimal level of the income tax \( \tau^* \) should recognize the distribution of the welfare costs of the commodity tax (which also tends to reduce the equilibrium level of \( \bar{t} \)).

Second, the aggregate elasticity of labor with respect to the commodity tax \( \varepsilon_{\ell-t}^A \) and the aggregate elasticity of consumption with respect to the labor income tax, \( \varepsilon_{c-t}^A \), affect the optimal tax structure \( \bar{t} \) and \( \tau^* \) in different ways: i) If \( \varepsilon_{\ell-t}^A > 0 \) and \( \varepsilon_{c-t}^A > 0 \), then increases in these elasticities increase the marginal tax revenue of both the commodity and labor income taxes which tends to lead to reductions of the equilibrium tax rates \( \bar{t} \) and \( \tau^* \). ii) If \( \varepsilon_{\ell-t}^A > 0 \) and \( \varepsilon_{c-t}^A > 0 \), then reductions in any of these elasticities reduce the weight \( \theta \) in conditions (30) and (31) which increases the influence of the typical determinants of the income tax in the optimal level of the commodity tax \( \tau^* \). That is, the aggregate labor income, the socially weighted aggregate labor income and the efficiency costs from labor income taxation become more relevant to determine \( \bar{t} \). Similarly, for \( \varepsilon_{\ell-t}^A > 0 \) and \( \varepsilon_{c-t}^A > 0 \), reductions in \( \varepsilon_{\ell-t}^A \) and \( \varepsilon_{c-t}^A \) increase the influence of the typical determinants of the commodity tax (that is, the aggregate consumption, the weighted aggregate consumption and the inefficiency costs from commodity taxation) on the optimal level of the labor income tax \( \tau^* \) (we prove these results in Proposition 3).

Third, there are values of the consumption-commodity tax, labor supply-commodity tax, supply of labor-income tax and consumption-income tax elasticities in which \( \theta \rightarrow 0 \) which implies that the main determinants of the design of the commodity tax \( \tau^* \) are the typical determinants of a labor income tax (those characterized in condition (12) in Proposition 2). In other words, for the limiting case of \( \theta \rightarrow 0 \), the aggregate level of consumption, the weighted tax burdens from consumption and the efficiency costs of commodity taxes should play a marginal role in determining the optimal level of the commodity tax (we show this result in Proposition 4).

Simultaneously, for \( \theta \rightarrow 0 \) the aggregate level of labor income, the socially aggregated level of income, and the elasticity of labor with respect to the income tax should play a marginal role in determining the optimal level of \( \tau^* \). Instead, the main determinants of an income tax \( \tau^* \) should be the typical determinants of a commodity tax, that is, the aggregate level of consumption, a socially aggregated level of consumption, and the price-elasticity of consumption (we also show this result in Proposition 4).

On what follows, Proposition 3 identifies the ways how changes in the aggregate elasticities characterized in (7), (8), (9) and (10) affect the relative weight \( \theta \) in the design of \( t^* \) and \( \tau^* \), and proposition 4 identifies the sufficient conditions for the limiting case in which \( \theta \rightarrow 0 \).

**Proposition 3.** If \( \varepsilon_{\ell-t}^A > 0 \) and \( \{\varepsilon_{\ell-t}^A \varepsilon_{c-t} - \varepsilon_{c-t}^A \varepsilon_{\ell-t}^A\} > 0 \), then

1. \( \text{sign}(\varepsilon_{\ell-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{\ell-t}^A) \),
2. \( \text{sign}(\varepsilon_{c-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{c-t}^A) \).
3. \( \forall \theta \in (0,1), \text{sign}(\varepsilon_{\ell-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{\ell-t}^A) \) and \( \forall \theta > 1, -\text{sign}(\varepsilon_{\ell-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{\ell-t}^A) \).
4. \( \forall \theta \in (0,1), \text{sign}(\varepsilon_{c-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{c-t}^A) \) and \( \forall \theta > 1, -\text{sign}(\varepsilon_{c-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{c-t}^A) \).

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8 By welfare costs we mean the loss in the indirect utility of households because of the price and negative income effects caused by an increase in commodity and labor income taxes.

9 Although we don’t prove formally this result, this outcome is easily observed from the first order conditions (21) and (22) of proposition 2.
Proof: For results (3.i) and (3.ii), recall \( \theta = \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A / \{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \} \), hence \( \frac{d\theta}{d\varepsilon_{\ell-t}^A} = \frac{(\varepsilon_{C-t}^A)^2 \varepsilon_{\ell-t}^A}{\{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \}^2} \) and \( \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A > 0 \land \{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \} > 0 \) implies \( \text{sign}(\varepsilon_{\ell-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{\ell-t}^A) \). Similarly, \( \frac{d\theta}{d\varepsilon_{C-t}^A} = \frac{\varepsilon_{\ell-t}^A (\varepsilon_{\ell-t}^A)^2}{\{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \}^2} \), hence \( \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A > 0 \) and \( \{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \} > 0 \) implies \( \text{sign}(\varepsilon-C-t\varepsilon-C-tA) \Rightarrow \text{sign}(d\theta/d\varepsilon_{C-t}^A) \).

For results (3.iii) and (3.iv), \( \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A > 0 \) and \( \{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \} > 0 \) \( \Rightarrow \theta > 0 \). It is simple to show that \( \frac{d\theta}{d\varepsilon_{C-t}^A} = \frac{\varepsilon_{C-t}^A (1-\varepsilon_{C-t}^A)}{\{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \}} \) for \( \forall \theta \in (0,1) \), \( \text{sign}(\varepsilon_{C-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{C-t}^A) \) and \( \forall \theta > 1, \text{sign}(\varepsilon_{C-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{C-t}^A) \). Similarly, \( \frac{d\theta}{d\varepsilon_{\ell-t}^A} = \frac{\varepsilon_{C-t}^A (1-\varepsilon_{C-t}^A)}{\{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \}} \) for \( \forall \theta \in (0,1) \), \( \text{sign}(\varepsilon_{\ell-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{\ell-t}^A) \) and \( \forall \theta > 1, \text{sign}(\varepsilon_{\ell-t}^A) \Rightarrow \text{sign}(d\theta/d\varepsilon_{\ell-t}^A) \). #

As we mentioned before, Proposition 3 identifies conditions in which changes in the direct and cross price elasticities with respect to the commodity and income taxes can lead to changes in \( \theta \). Of particular interest is the case in which changes in the direct and cross price elasticities lead to a fall of \( \theta \) and modify the relative importance of the distribution of welfare costs and efficiency considerations from commodity and income taxation in the determination of \( t^* \) and \( \tau^* \). Recall a fall in \( \theta \) means that \( t^* \) depends more heavily on the benchmark determinants of \( t^* \) (that is, the aggregate level of income, the socially weighted aggregate level of labor income and the weighted aggregate elasticity of the supply of labor and income taxes identified in condition 12 of Proposition 1) instead of the typical determinants of \( t^* \) (the aggregate level of consumption, the socially weighted aggregate level of consumption and the weighted aggregate elasticity of the consumption and the commodity tax). A similar interpretation is given to the impact of a fall of \( \theta \) in \( \tau^* \).

**Proposition 4.** Assume \( \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A > 0 \) and \( \{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \} > 0 \), then \( \exists \varepsilon_{C-t}^A, \varepsilon_{\ell-t}^A, \varepsilon_{C-t}^A, \varepsilon_{\ell-t}^A : \theta \rightarrow 0 \) which implies

\[
\begin{align*}
t^* & \equiv \left( \frac{\varepsilon_{\ell-t}^A}{\varepsilon_{C-t}^A} \right) \bar{t}^* \\
\tau^* & \equiv \left( \frac{\varepsilon_{C-t}^A}{\varepsilon_{\ell-t}^A} \right) \bar{t}^*
\end{align*}
\]

**Proof:** By Proposition 3, \( \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A > 0 \) and \( \{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \} > 0 \Rightarrow \theta > 0 \). Since \( \theta = \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A / \{ \varepsilon_{C-t}^A \varepsilon_{\ell-t}^A - \varepsilon_{\ell-t}^A \varepsilon_{C-t}^A \} \), \( \exists \varepsilon_{C-t}^A, \varepsilon_{\ell-t}^A, \varepsilon_{C-t}^A, \varepsilon_{\ell-t}^A : \theta \rightarrow 0 \). By conditions (18) and (19) of Proposition 2, \( t^* \equiv \left( \frac{\varepsilon_{\ell-t}^A}{\varepsilon_{C-t}^A} \right) \bar{t}^* \) and \( \tau^* \equiv \left( \frac{\varepsilon_{C-t}^A}{\varepsilon_{\ell-t}^A} \right) \bar{t}^* \). #

Proposition 4 shows the limiting case \( \theta \rightarrow 0 \), where the determinants of the design of the commodity tax \( t^* \) are the benchmark or typical determinants of a labor income tax, and, simultaneously, the main determinants of an income tax \( \tau^* \) are the typical determinants of a commodity tax.

An interesting implication of Proposition 4 for the optimal tax structure is that even when the statutory definition of the commodity tax base \( t^* \) is defined by aggregate consumption in our
economy is the broad tax base, the level of $t^*$ is determined as if the commodity tax were collecting tax revenue from the aggregate level of labor income which is the narrow tax base (recall that consumption is determined by the full income in the economy and therefore consumption and full income are the broadest definitions of a tax base, while the income tax is applied to labor income which is a narrower definition of a tax base).

In this case this effect tends to limit the size of the commodity tax on the optimal tax structure (in other words, this effect tends to imply $t^* < \bar{t}^*$). The opposite occurs with $\tau^*$. Despite that the statutory definition of the tax base of $\tau^*$ is narrowly defined (i.e. the tax base $\tau^*$ is defined over the aggregate labor income), the level of $\tau^*$ is determined as if the labor income tax were collecting tax revenue from a broad tax base (that is, from the aggregate consumption). In this case this effect tends to increase the size of $\tau^*$ on the optimal tax structure (this effect tends to imply $\tau^* > \bar{\tau}^*$).

4. Policy Implications

In this section we discuss some of the policy implications of propositions 2, 3 and 4. First, our analysis characterize conditions to identify the main determinants of the design of both the commodity and income tax. These determinants change depending on the signs and values of the direct and cross price elasticities. A benchmark result of the taxation literature suggests that the aggregate level of consumption, the weighted tax burdens from consumption and the efficiency costs of commodity taxes are the main determinants of a commodity tax. Our analysis suggests additional determinants such as the aggregate level of income, the socially weighted aggregate level of labor income and the weighted aggregate elasticity of the supply of labor and income. A similar conclusion is obtained for the determinants of the income tax.

Second, we also identify conditions for a limiting case (see proposition 4) in which the main determinants of the commodity and income tax (identified in the benchmark case) should play only a marginal role in the design of these taxes. Third, another interesting implication of this limiting case is that even when the statutory definition of the commodity tax base $t^*$ is defined by aggregate consumption, which in our economy is the broad tax base, the level of $t^*$ should be determined as if the commodity tax were collecting tax revenue from the aggregate level of labor income which is the narrow tax base in this economy. These results suggests a limit to the use of the commodity tax and a rationale to increase the size of the income tax in the tax structure.

5. Conclusion

We develop a simple model of optimal taxation to characterize the optimal structure of a commodity tax and a labor income tax system for an economy with international inter-family transfers. Our model produces new insights about the main determinants of the design of the optimal level of the commodity and labor income tax. In particular, we find that if both the aggregate elasticity of labor and the commodity tax, and the aggregate elasticity of consumption and the labor income tax are not zero, then the optimal commodity tax is characterized by a linear combination of the determinants of a commodity tax and a labor income tax. Similarly, the optimal income tax is characterized by a linear combination of a commodity tax and a labor income tax. These results have implications for how equity and efficiency considerations determine, respectively, the commodity and the income tax.

The main result from this paper is that there are values of direct and cross price elasticities in which the main determinants of the design of a commodity tax are, actually, the typical determinants of a labor income tax (i.e., the commodity tax is mainly explained by the aggregate
level of labor income, the socially aggregated level of income, and the elasticity of labor with respect to the income tax). This outcome also implies that the aggregate level of consumption, the weighted tax burdens from consumption and the efficiency costs of commodity taxes (considered by the literature as the main determinants of a commodity tax) might play only a marginal role in determining the optimal level of the commodity tax.

These conditions in the direct and cross price elasticities mentioned above, also imply that the aggregate level of labor income, the socially aggregated level of income, and the elasticity of labor with respect to the income tax (considered by the literature as the main determinants of a linear income tax) could play just a marginal role in determining the optimal level of an income tax. Instead, the main determinants of the income tax should be the typical determinants of a commodity tax, that is, the aggregate level of consumption, the socially aggregated level of consumption, and the price-elasticity of consumption. To the best of our knowledge, these results have not being recognized in the literature.

Finally, these results have interesting implications for the design of the tax system. In particular, our results imply that even when the statutory definition of the commodity tax base is given by the aggregate level of consumption, which in our economy is the broadest definition of a tax base, the design of the level of the optimal commodity tax is actually determined as if a commodity tax were collecting tax revenue from the aggregate level of labor income which in this economy is the narrow tax base available for tax policy makers. In this case this effect tends to limit the size of the commodity tax on the optimal tax structure.

This outcome also means that the opposite occurs with the optimal income tax. In other words, despite that the statutory definition of the tax base of the income tax is narrowly defined (i.e. the tax base of the income tax is defined over the aggregate labor income), the level of the optimal income tax is determined as if the labor income tax were collecting tax revenue from a broad tax base (that is, from the aggregate level of consumption in the economy). This effect tends to increase the size of the optimal income tax on the optimal tax structure.

Finally, it is relevant to point out that the main limitation of this paper is that it considers a benevolent social planner and it ignores political constraints (such as the need to form legislative coalitions in congress) that are relevant for tax policy making. Future work on this issue should address this issue.

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