

## The Squared Coefficient of Variation as an Inequality Index: A Social Evaluation Characterization

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**Abstract:** The squared coefficient of variation ( $C^2$ ) is a well-known index of relative inequality. However, the existing economic theory of inequality does not contain a characterization of this index in terms of the properties of the underlying social evaluation function. It is well-known that if the Atkinson-Kolm-Sen (AKS) index of relative inequality derived from a social evaluation relation defined on a space of distributions is  $C^2$ , it is necessary that the relation satisfies a ‘transfer neutrality’ condition.

This paper obtains a complete characterization of the index by proving the converse. It is shown that, in the presence of other standard assumptions on the social evaluation relation, transfer neutrality implies a particular social evaluation function and that the corresponding AKS relative inequality index coincides with  $C^2$ . This inequality index and the corresponding social evaluation function is then applied in a relatively unexplored area of empirical research. It is shown that in India *inequality* (as measured by  $C^2$ ) in the distribution of a variable that indicates the width of accessibility of bank credit *increased* in the ten or so years following the introduction of economic reforms in the early 1990s. At the same time there was an *increase* in the *average* value of this indicator. The question, therefore, arises as to the direction of change of over-all social welfare from this particular attribute. The social evaluation underlying  $C^2$  implies that, on balance, there was a decline in the welfare of the country in this respect.

**Keywords:** Coefficient of variation, Social evaluation, Transfer neutrality, Bank credit

**JEL Classifications:** D63, D31, I30, C69

### 1. Introduction

The social evaluation approach to the measurement of inequality developed by Atkinson (1970), Kolm (1969) and Sen (1997) has been used in the literature to develop various classes of inequality indices. Axioms imposed on the social evaluation relation yield a particular representation of this relation viz. the ‘equally distributed equivalent income’ function which is then used to define inequality indices. The index of *relative* inequality obtained in this way is often called the Atkinson-Kolm -Sen (AKS) inequality index corresponding to the given social evaluation relation.

It is known that the procedure can be reversed: we can start with specific inequality indices and recover the social value judgements underlying the indices. (See Blackorby and Donaldson,

1978; and Chakravarty, 1990). This has made possible the analysis of social values implicit in specific inequality indices.

In this paper we shall be concerned with the inequality index given by the squared coefficient of variation ( $C^2$ ). (The coefficient of variation,  $C$ , of a statistical distribution is defined to be its standard deviation divided by the arithmetic mean.) Its simplicity is one of the factors behind its wide use in practice. Among recent empirical contributions making use of  $C^2$  (in addition to other indices) are Decoster and Schokkaert (2002), Stewart et. al. (2005) and Sala-i-Martin (2002).

This paper has a theoretical part and an empirical part. The theoretical part obtains a complete social evaluation characterization of  $C^2$  as an index of relative inequality. It is well-known that the social evaluation relation underlying  $C^2$  has a ‘transfer neutrality’ property. This paper shows that, in the presence of some standard and widely used assumptions on the social evaluation relation, the converse is also true: transfer neutrality uniquely implies a particular class of social evaluation relations and that the corresponding A-K-S relative inequality index turns out to be  $C^2$ . In this sense it completes the proof that this property is essentially the defining characteristic of the social values underlying  $C^2$ . It may be noted that characterization of  $C^2$  in terms of axioms imposed on an inequality index is available in the literature. In contrast, the focus in this paper is on obtaining a characterization in terms of the underlying social evaluation.

The empirical part of the paper applies the social evaluation underlying  $C^2$  in an area of research which seems to have attracted little attention in the literature so far. We consider the distribution of a variable that measures the width of accessibility of bank credit in India. It is shown that, over a period of about a decade after the introduction of economic reforms in India in the early 1990s, there was a decline in the social welfare that the country derived from this particular attribute.

Section 2 below introduces the notations, definitions and the axioms. Section 3 contains the theoretical results. Section 4 reports the empirical finding. Section 5 concludes the paper.

## 2. Notations, Definitions and Axioms

Let  $n$  denote the number of individuals in a society.  $N = \{1, 2, \dots, n\}$  is the set of individuals. Let  $n \geq 2$ .

A *distribution* is an  $n$ -vector  $x = (x_1, x_2, \dots, x_n)$  where  $x_i$  is the amount of the attribute allocated to individual  $i$ ,  $i = 1, 2, \dots, n$ .  $X$  is the set of all non-negative non-zero distributions. In this and the following Sections it will be assumed for convenience that the attribute represents income.  $x^*$  will denote the rearrangement of  $x$  in non-increasing order. For all  $x$  in  $X$ ,  $\mu(x)$  will denote arithmetic mean of  $x$ . An  $x$  in  $X$  is called an *equal distribution* if  $x = \mu(x) \cdot 1_n$ .

For any  $n$ -vector  $a = (a_1, a_2, \dots, a_n)$ ,  $a_{-i}$  denotes the  $(n-1)$ -vector  $(a_1, a_2, \dots, a_{i-1}, a_{i+1}, \dots, a_n)$ .

A social evaluation relation is a binary relation  $R$  on  $X$ . For all  $x, y$  in  $X$ ,  $x R y$  will be interpreted to mean that  $x$  is socially weakly preferred to  $y$ .  $P$  and  $I$  will denote the asymmetric and the symmetric components of  $R$  respectively. A real-valued function on  $X$  representing  $R$  will be called a *social evaluation function* corresponding to  $R$ .

A Pigou-Dalton transfer is an income transfer from a richer to a poorer person by an amount less than their income difference. For simplicity it is assumed that Pigou-Dalton transfers do not

change the relative rankings of the persons between whom the transfer is made i.e. if  $x$  in  $X$  is such that  $x_i > x_j$  and  $y$  in  $X$  is such that  $y_i = x_i - \Delta$ ,  $y_j = x_j + \Delta$  and  $y_k = x_k$  for all  $k \in N - \{i, j\}$  where  $\Delta$  is such that  $0 < \Delta < (x_i - x_j)/2$ ,  $y$  is said to have been obtained from  $x$  by a Pigou-Dalton transfer. In this paper rank-preserving Pigou-Dalton transfers will be simply called ‘transfers’.

For a given social evaluation relation  $R$  on  $X$ , the *equally-distributed-equivalent income* (EEDI) associated with a distribution  $x$ ,  $E_R(x)$ , is the amount of the attribute such that if each individual is allocated this amount, the society is indifferent between the resulting equal distribution and the distribution  $x$  i.e. for any  $x$  in  $X$ ,  $E_R(x)$  is such that  $E_R(x).1_n I x$ . The EEDI function for  $R$  is the mapping  $E_R: X \rightarrow \Re$  which assigns to each  $x$  in  $X$  the associated EEDI. Under suitable conditions on  $R$  the function  $E_R$  is well-defined and represents  $R$ . The Atkinson-Kolm-Sen inequality index corresponding to  $R$  is the mapping  $I_R: X \rightarrow \Re$  for which, for all  $x$  in  $X$ ,

$$I_R(x) = 1 - (E_R(x)/\mu(x)).$$

$I_R$  is a relative inequality index (i.e. it is homogeneous of degree 0). It lies between 0 and 1, and is 0 if and only if  $x$  is an equal distribution.

The following axioms are imposed on  $R$ .

- (1) **Ordering (ORD):**  $R$  is an ordering (i.e. a reflexive, complete and transitive relation) on  $X$ .
- (2) **Continuity (CONT):** For all  $x$  in  $X$ , the sets  $\{y \in X : yPx\}$  and  $\{y \in X : xPy\}$  are open.
- (3) **Monotonicity under Equality (ME):** For all *equal distributions*  $x$  and  $y$  in  $X$  such that  $x > y$ ,  $xPy$ .
- (4) **Weak Monotonicity (WM):** For all  $x$  and  $y$  in  $X$  such that  $x \geq y$ ,  $xRy$ .
- (5) **Anonymity (ANON):** For all  $x$  and  $y$  in  $X$  such that  $x$  is a permutation of  $y$ ,  $xIy$ .
- (6) **Population Replication Invariance (PRI):** If  $x$  and  $y$  in  $X$  are such that  $y$  is obtained by a  $k$ -fold replication of the population in  $x$  for some positive integer  $k$  [i.e., for all  $p$  in  $N$ ,  $x_p = y_p = y_{n+p} = \dots = y_{n(k-1)+p}$ ], then  $x I y$ .
- (7) **Transfer Principle (TRANS):** If  $x$  and  $y$  in  $X$  are such that  $x$  is obtained by a finite sequence of transfers from  $y$ , then  $xPy$ .
- (8) **Homotheticity (HOM):** For all  $x$  and  $y$  in  $X$  and for all positive scalars  $\lambda$ ,  $xRy$  if and only if  $\lambda x R \lambda y$ .
- (9) **Transfer Neutrality (TN):** Let  $z \in X$ . Let  $x, y$  in  $X$  and a scalar  $\Delta$  be such that, for some  $i, j, k, l$  in  $N$  for which  $z_i > z_j$ ,  $z_k > z_l$  and  $\{i, j\} \neq \{k, l\}$ ,
  - (i)  $0 < \Delta < \min\{(z_i - z_j)/2, (z_k - z_l)/2\}$ ;
  - (ii)  $x_i = z_i - \Delta$ ,  $x_j = z_j + \Delta$  and  $x_p = z_p$  for all  $p \in N - \{i, j\}$ ; and
  - (iii)  $y_k = z_k - \Delta$ ,  $y_l = z_l + \Delta$  and  $y_q = z_q$  for all  $q \in N - \{k, l\}$ .
 Then  $[xIy \Leftrightarrow z_i - z_j = z_k - z_l]$ .

Regarding the above nine axioms, **ORD**, **CONT**, **ANON**, **TRANS**, **PRI** and **HOM** are widely used common conditions and hardly need any comments.

**ME** and **WM** together constitute weaker requirement than Monotonicity which says that for all  $x$  and  $y$  in  $X$  such that  $x \geq y$  and  $x \neq y$ ,  $x P y$ . The Monotonicity condition itself is widely used in the normative literature on inequality. However, it requires that in the society's judgement any increase in inequality resulting from an increase in one individual's income when all other incomes are held constant are overbalanced by the resulting increase in total income. While this may be the case for specific distributions, imposing this condition on the social evaluation relation as a prior requirement does not seem to be warranted.

Weakening of the Monotonicity condition is also motivated by the fact that properties of the social evaluation relation often lead to corresponding properties of the resulting inequality index. Let an inequality index  $I$ , a mapping from  $X$  into the real line, be called monotonic if, for all  $x$  and  $y$  in  $X$ ,  $[x_1^* > y_1^* \text{ and } x_i^* = y_i^* \text{ for all } i \text{ in } N - \{1\}]$  implies  $I(x) > I(y)$ . The coefficient of variation, the subject of this paper, is not monotonic on the space of non-negative non-zero income distributions. (It will, however, be monotonic on the space of positive distributions.)

One of the principal roles played by monotonicity-type conditions is in proving the existence of EDEI functions. Conditions weaker than Monotonicity have been used for this purpose in the inequality literature before. For example, in his axiomatic derivation of the generalized Gini index Weymark (1981) combines **ME** with the additional assumptions that the social evaluation relation is represented by a social welfare function  $W$  and that each level surface of  $W$  crosses the line of equality. However, these additional assumptions appear to be tantamount to directly assuming the existence of an EDEI function. The same remark applies to the approach taken in Chakravarty (1990, Ch.2). While the combination of **ME** and **WM** is technically stronger, it retains enough intuitive transparency without demanding full monotonicity.

**TN** is a formal statement of the transfer neutrality property mentioned in the Introduction. It requires that, starting from any given distribution, if an equal amount of transfer takes place between two different pairs of individuals, the society is indifferent between the two resulting distributions if and only if, in the initially given distribution, the income difference was the same for the two pairs of individuals. (See, in this connection, the 'transfer sensitivity' literature, for instance, Shorrocks and Foster (1987). In the context of measurement of social deprivation Chakraborty, Pattanaik and Xu (2008) analyzed a closely related axiom called 'equivalent transfer' on the deprivation index.) Since the condition does not specify the pattern of preference between the initial distribution and either of the two resulting distributions, it does not imply **TRANS**.

### 3. The Characterization Result

This Section shows that the axioms on the social evaluation relation  $R$  introduced in Section 2 imply and are implied by a particular EDEI representation. Subsequently it is seen that the AKS relative inequality index corresponding to this representations coincides with  $C^2$ .

**Proposition 1:** The social evaluation relation  $R$  on  $X$  satisfies **ORD**, **CONT**, **ME**, **WM**, **ANON**, **PRI**, **TRANS**, **HOM** and **TN** if and only if it is represented by a mapping  $E^C: X \rightarrow \mathfrak{R}$  such that, for all  $x$  in  $X$ ,

$$E^C(x) = 2\mu(x) - (1/n) \left[ \sum_{i=1}^n (x_i)^2 / \mu(x) \right].$$

**Proof:** To prove the ‘only if’ part, it is first established that **ORD**, **CONT**, **ME**, **WM** and **ANON** imply that, for any  $x$  in  $X$ , there exists a vector  $w(x^*) = \{w_1(x^*), w_2(x^*), \dots, w_n(x^*)\} \in \{w \in \mathfrak{R}_+^n: \sum_{i=1}^n w_i = 1\} = W$  (say) such that

$$(w(x^*).x^*)1_n = \left( \sum_{i=1}^n w_i(x^*)x_i^* \right).1_n I x \quad (1)$$

If  $x$  is such that  $x = \mu(x).1_n$ , then for any  $w$  in  $W$ ,  $(w.x^*).1_n = x I x$  since  $R$  satisfies **ORD** and is, therefore, reflexive. If not, then  $x_1^* > x_n^*$  so that for  $(w_1=1, w_2=w_3=\dots=w_n=0)$ ,  $(w.x^*).1_n = x_1^*.1_n R x^*$  by **WM**. Hence, by **ANON**,  $(w.x^*).1_n R x$ . Now, if  $(w.x^*).1_n I x$ ,  $w$  satisfies (1). If not, then

$$(w.x^*).1_n P x \quad (2)$$

On the other hand, for  $(w_1=w_2=\dots=w_{n-1}=0, w_n=1)$ , **ANON** and **WM** imply  $x I x^* R (w.x^*).1_n$ . If  $x I (w.x^*).1_n$ , then  $w$  satisfies (1). Otherwise,

$$x P (w.x^*).1_n \quad (3)$$

Existence of a  $w(x^*)$  satisfying (1) now follows from (2), (3) and the facts that  $R$  satisfies **CONT** and that  $w$  can be varied continuously over  $W$ .

Let the mapping  $E^C: X \rightarrow \mathfrak{R}$  be such that, for all  $x$  in  $X$ ,

$$E^C(x) = w(x^*).x^* \quad (4)$$

where  $w(x^*)$  is defined by equation (1).

It can be verified that, for any  $x$  in  $X$  and any  $w(x^*)$  and  $w(x^*)$  satisfying eqn. (1), **ORD**, **ME** and eqn. (1) imply that  $w(x^*).x^* = w(x^*).x^*$ . Hence, the mapping  $E^C$  is well-defined. Trivially,  $E^C(x) = E^C(x^*)$ . **ORD** and **ME** imply that  $w(x^*).x^*$  represents  $R$ . Thus, the function  $E^C$  can be taken to be the social welfare function. However, since  $R$  is an ordinal preference relation,  $E^C$  will have only ordinal significance.

Since  $R$  satisfies **HOM**,  $E^C(x)$  is homogeneous of degree one at all  $x$  in  $X$ . Hence, for any  $x$  in  $X$ , for all  $w(x^*)$  satisfying (1) and for all  $i = 1, 2, \dots, n$ ,  $w_i(x^*)$  is homogeneous of degree zero.

For the subsequent argument, the functions  $w_i$  are written in an alternative form. It can be verified that there exist mappings  $v_i$ ,  $i = 1, 2, \dots, n$ , from  $\{(x^*/\mu(x))_{-i} : x \in X\}$  into  $[0, 1]$  such that for all  $x$  in  $X$  and  $w(x^*)$  defined in (1),  $w_i(x^*) = v_i[(x^*/\mu(x))_{-i}]$ .

However, **ANON** implies that, for all  $x$  in  $X$  and for all  $i = 1, 2, \dots, n$ ,  $w_i(x^*) = v[(x^*/\mu(x))_{-i}]$  for some function  $v$  and that  $v$  is symmetric in its arguments.

**TN** implies that  $v$  is an affine function. If  $v$  has a non-affine form, then, recalling that  $X$  includes all non-negative non-zero distributions, it is possible to choose  $z, x, y$  in  $X$  and a scalar  $\Delta > 0$  such that the antecedent in the statement of **TN** is satisfied but  $E^C(x)$  and  $E^C(y)$ , as given by (4), are unequal i.e.  $\neg(xIy)$ . Hence, **TN** is violated.

Therefore, there exist some scalar  $\alpha$  and some  $(n-1)$ -vector  $\beta_{n-1}$  such that for all  $i = 1, 2, \dots, n$ ,

$$v[(x^*/\mu(x))_{-i}] = \alpha + \beta_{n-1} \cdot (x^*/\mu(x))_{-i} \quad (5)$$

However, since  $v$  is symmetric, the components of  $\beta_{n-1}$  equal some scalar  $\beta$ . Thus, for all  $x$  in  $X$  and for all  $i$  in  $N$ ,

$$w_i(x^*) = v[(x^*/\mu(x))_{-i}] = \alpha + \beta \sum_{j \neq i} (x_j^*/\mu(x)) = \alpha + \beta n - \beta(x_i^*/\mu(x)).$$

It is easily verified that **TRANS** implies that  $\beta > 0$ . Therefore, (4) implies

$$E^C(x) = \sum_{i=1}^n [\alpha + \beta n - \beta(x_i^*/\mu(x))] x_i^* \quad \text{for all } x \text{ in } X \quad (6)$$

for some scalar  $\alpha$  and some scalar  $\beta > 0$  such that

$$\alpha + \beta n - \beta x_i^*/\mu(x) \geq 0 \quad \text{for all } i = 1, 2, \dots, n \quad (7a)$$

$$\text{and} \quad \sum_{i=1}^n (\alpha + \beta n - \beta x_i^*/\mu(x)) = 1 \quad (7b)$$

Let  $\lambda = \alpha + \beta n$ . Then by equation (7b), we get  $\lambda = \beta + 1/n$ . Hence Equation (6) implies

$$E^C(x) = \sum_{i=1}^n [\lambda - \beta(x_i^*/\mu(x))] x_i^* = \beta \mu n + \mu - \beta \left[ \sum_{i=1}^n (x_i^*)^2 / \mu(x) \right] \quad \text{for some } \beta > 0$$

We now invoke **PRI**. It could be verified that this condition requires that  $\beta = 1/n$ . Thus, for all  $x$  in  $X$ ,

$$E^C(x) = 2\mu - (1/n) \left[ \sum_{i=1}^n (x_i^*)^2 / \mu(x) \right] \quad (8)$$

This completes the proof of the ‘only if’ part of Proposition 1.

The ‘if’ part is easily verified. Q.E.D.

It is now easily seen that the Atkinson-Kolm-Sen (AKS) inequality index corresponding to a social evaluation relation  $R$  satisfying the axioms stated in Proposition 1 above, is given by the mapping  $I: X \rightarrow \mathfrak{R}$  for which, for all  $x$  in  $X$ ,

$$I^{\beta}(x) = \sum_{i=1}^n (x_i/\mu(x))^2 - 1.$$

It is readily seen that the right hand side of the last equality is nothing but  $C^2$ , the squared coefficient of variation. Since all of the axioms on the social evaluation relation  $R$  considered in this paper, other than **TN**, are standard assumptions, the social evaluation characterization of  $C^2$  presented here leads to the conclusion that **TN** is essentially the defining characteristic of the social values underlying this inequality index. Our work, thus, leads to a fuller understanding of this index than what can be inferred from the existing literature.

Before leaving this theoretical discussion, it may be noted that the ordinal social welfare function  $E^C$  obtained in Eqn. (8) above can be rewritten as follows: For all  $x$  in  $X$ ,

$$E^C(x) = \mu(x)[2 - (1/n) \sum_{i=1}^n (x_i/\mu(x))^2] = \mu(x)[1 - \{(1/n) \sum_{i=1}^n (x_i/\mu(x))^2 - 1\}] = \mu(x)[1 - C^2(x)]$$

Thus, for any  $x$  in  $X$ , the value  $E^C(x)$  of the ordinal social welfare function has a simple interpretation: it is the arithmetic mean of  $x$  “corrected for” inequality as measured by  $C^2$ . Therefore,  $E^C$  is the  $C^2$ -based ordinal social welfare function.

#### 4. An Empirical Application: Social Welfare from Width of Accessibility of Bank Credit in India

In the preceding Sections we considered the squared coefficient of variation as an index of inequality of *income* distribution. This was, however, purely a matter of convenience. In place of income we can consider any quantifiable attribute. Moreover, what we called an “individual” can be any *unit of observation* (for instance, households, firms, regions, countries etc.); and what we called a “society” can be any group of such units of observation. The simple social welfare function stated at the end of the preceding Section is, therefore, quite versatile in its applications.

As an illustration of this versatility, in this Section we report the results of applying this function to an area of research that seems to have attracted little attention so far in the literature. We consider the problem of determining the direction of change in social welfare derived from the width of accessibility of bank credit in India in the decade following the introduction of economic reforms in the first half of the 1990’s.

The importance of credit supply in a developing economy such as India hardly needs explanation. Credit supply, however, is a multi-faceted concept. We shall concentrate on one of its aspects. We distinguish between *credit deepening* and *credit widening* and focus our attention on the latter. Credit deepening is related to increasing the per capita availability of credit. Credit widening, on the other hand, refers to the extending horizontal spread of the benefits of credit among the population. We shall confine ourselves to *bank* credit since banks constitute the major source of credit in India. In our work the degree of bank credit widening is measured by the *number of outstanding credit accounts in the scheduled commercial banks per capita* (henceforth, ‘number of credit accounts per capita’ or NCAPC).

We shall compare the social welfare (accruing to the country as a whole) from the distribution of NCAPC in the year 1994 with that in 2005. For that purpose we need to compute the values of



the arithmetic mean and the squared coefficient of variation of the distributions of NCAPC for these two years. The population distributions of this variable, however, are not available. What we have is the distribution across the states of the country. Under these circumstances we follow the customary procedure of ignoring the intra-state inequalities.

Moreover, data for these years are available for only 17 states. However, these are the major states in terms of population size. Together they constituted more than 95 per cent of the total population of India in both of the years.

For each of the 17 states and for the two years of our interest Table 1 displays the value of NCAPC and share of the state's population in the total population of these states. Figures for NCAPC are as in end-March of the relevant year. The population share figures are obtained by interpolations based on the data in the Census of India, 1991, 2001 and 2011. For 2005 the figure for the state of Bihar includes that for Jharkhand, the state created after 1994 by carving out a part of Bihar. For similar reasons the figures for 2005 for the states of Madhya Pradesh and Uttar Pradesh include those for Chhattisgarh and Uttaranchal respectively.

**Table 1.** NCAPCs and population shares of 17 states of India, 1994 and 2005

State	NCAPC		Share of state population in Total population	
	1994	2005	1994	2005
1. Andhra Pradesh	0.103	0.118	0.08	0.07
2. Assam	0.042	0.030	0.03	0.03
3. Bihar	0.058	0.032	0.11	0.11
4. Gujarat	0.046	0.043	0.05	0.05
5. Haryana	0.064	0.052	0.02	0.02
6. Himachal Pradesh	0.055	0.063	0.01	0.01
7. Jammu and Kashmir	0.046	0.037	0.01	0.01
8. Karnataka	0.098	0.129	0.05	0.05
9. Kerala	0.126	0.141	0.03	0.03
10. Madhya Pradesh	0.049	0.037	0.08	0.08
11. Maharashtra	0.052	0.086	0.10	0.10
12. Orissa	0.089	0.067	0.04	0.04
13. Punjab	0.070	0.066	0.03	0.02
14. Rajasthan	0.047	0.043	0.05	0.06
15. Tamil Nadu	0.110	0.195	0.07	0.06
16. Uttar Pradesh	0.048	0.044	0.17	0.18
17. West Bengal	0.063	0.044	0.08	0.08
<b>Source:</b> Reserve Bank of India (1996 and 2006) and interpolations based on Government of India (1992, 2002 and 2012).				

Under our assumption regarding the absence of intra-state disparities, Table 1 describes the relative frequency distribution of NCAPC in India in the two years. This enables us to compute the mean  $\mu$ , the squared coefficient of variation  $C^2$ , and the social welfare for each of the years. These are reported in Table 2 on the next page.



**Table 2.** Social welfare from NCAPC in India in 1994 and 2005

Year	Mean ( $\mu$ )	Squared coefficient of variation ( $C^2$ )	Social welfare, $\mu(1 - C^2)$
1994	0.0665	0.1324	0.0577
2005	0.0688	0.4257	0.0395

It is shown that the value of  $C^2$  increased from 0.1324 in 1994 to 0.4257 in 2005. Thus, inequality in the distribution of NCAPC in India increased between these two years. However, as is seen from the table, the arithmetic mean too increased from 0.0665 to 0.0688 between the two years. What, then, do we conclude regarding the direction of change in the *over-all* benefit (accruing to the country as a whole) of credit widening? The improved efficiency in the task of widening the accessibility of bank credit has to be set against the increased inequality in its distribution. The social welfare function  $E^C$  (in the form stated at the end of previous Section) is used for this purpose. The values of the expression  $\mu(1 - C^2)$  for 1994 and 2005 are calculated to be 0.0577 and 0.0395 respectively. On balance, therefore, the country's welfare (as measured by the  $C^2$ -based social welfare function) from credit widening (as measured by NCAPC) declined between these two years.

## 5. Conclusion

In this paper we first obtained a complete social evaluation characterization of the inequality index given by  $C^2$ , the squared coefficient of variation. We imposed a number of axioms on the underlying social values (as described by the social evaluation relation on the space of the distributions of the attribute in question) and derived this index from these axioms. All of these axioms (excepting the axiom of Transfer Neutrality) are standard assumptions in the literature and are satisfied by all inequality indices. Therefore, we concluded that it is the Transfer Neutrality condition on the social values that is the defining characteristic of this particular index. In the process we also characterized the social welfare function that yields  $C^2$  as the Atkinson-Kolm-Sen inequality index.

The characterized inequality index is then applied to the task of investigating the question whether inequality in the distribution of the width of accessibility of bank credit in India increased or decreased in the first decade after economic reforms started in the early 1990's. We found this inequality to have increased. However, the picture was complicated by the fact that at the same time there was an increase in the average degree of credit widening (as measured by the arithmetic mean of the distribution). The social welfare function derived in the theoretical part of the paper was used for the purpose of coming to an over-all judgement. It was found that on balance social welfare from the benefit of credit widening (as indicated by this social welfare function) declined in India over the time period considered.

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