Abstract: Based on a traditional approach to the behavior of a bank which lends both private and public sector, and utilizing a typical expression for public debt accumulation, this paper concludes that the optimality of the number and size of banks depends heavily on the course of the public debt, *ceteris paribus*. If the intergenerational dimension of the public debt is assumed away, fiscal consolidation presupposes a limited number of banks under normal only profit, a sort of quasi-competitive banking. In the presence of intergenerational considerations, fiscal consideration requires a few efficient banks experiencing perhaps positive profit, which is consistent with the notion of workable competition. Consequently, the pre-consolidation size distribution of banks is immaterial policy-wise.

Keywords: Optimum number of banks, Public debt accumulation, Perfect vs. workable competition, Commercial bank seigniorage

JEL Classifications: E50, G20, L10

1. Introduction

What is the optimum number of banks? Certainly the one for which “after the structural characteristics of [the financial] market and the dynamic forces that shaped them have been thoroughly examined, there is no clearly indicated change that can be effected through public policy measures that would result in greater social gains than social losses”: This is what Markham (1950, p. 361) would have claimed quite persuasively as an operational norm for workable competition in the banking industry *à la* Clark (1940, 1961). The capitalist dynamics evolve around the growth of increasingly large lending clients eliciting the growth of big lenders beyond the smaller ones just because the latter cannot meet the increasing demand for loans.

Contrary to what theory predicts (see e.g. Worcester 1957 and Gowrisankaran and Holmes 2004) and evidence verifies for sectors outside banking (see e.g. White 1996 and Gowrisankaran and Holmes 2004), the coexistence of a few large banks with smaller ones, and even with a competitive fringe like in Germany, has come up as the “rule” rather than as a short-run phenomenon (see e.g. Coccorese 2002), because big banks come to cover the part of the loan market that the smaller ones would ration. Big banks can do so, because presumably of cost efficiencies absent from small banks and reflected in stock returns (see e.g. Kirkwood and Nahm 2006). Instead, small banks appear to be benefiting from moderate scope economies (see e.g. Allen and Liu 2007 and Berger and Humphrey 1993), which might be attributed to the better relationship banking of such banks (see e.g. Dijkstra 2013). Others, like Boot (2003), maintain that although scale and scope economies exist, in principle, they are difficult to attain in practice without further competition.

The big picture for the countries at least examined by these studies, namely Australia, Canada, United States, and Eurozone countries, is that the industry has stuck at some point on its downward
sloping part of its long-run cost curve. Responsible for this development have been *inter alia* the accumulation of impaired loans (insolvent temporarily or not borrowers, rescheduled loan agreements, and other past-due loans) and the adverse consequences of the 2008-2009 Lehman Brothers crisis, especially for the south of Eurozone. The subsequent public policy intervention favoring too-big-to-fail financial institutions and increased market concentration, is one which, as put by Dijkstra (2013, p. 4), prompts (a) artificial scale advantages by acting as de-facto insurance for such institutions and enabling thereby to borrow at lowers cost, and (b) scope economies for the mergers, leading to the possibility of monopoly rents. And, “Economies of scale and scope arising from market power and implicit too-big-to-fail subsidies might benefit individual institutions, but harm society as a whole.”

That is, the banking industry has not been anywhere close to workable competition and public policy does not appear to be addressing this problem properly. Nevertheless, note that this is a problem worsening with public debt accumulation, (United States, Southern Eurozone), because banks are among other things holders of such debt as well. In the next section, the optimality of the number and size of banks is put within this precisely context theoretically, and is found to depend heavily on the course of the public debt, *ceteris paribus*. From this point of view, and if the subjugation of a debt problem is considered to be welfare-enhancing, a public policy promoting a banking sector with a limited number of healthy banks is in the right direction. Section 3 concludes with a discussion of this policy in connection with the size of the banks and the political economy surrounding public policy.

### 2. Theoretical Considerations

#### 2.1 The Basic Model

The discussion of bank industry structure is made in the spirit of Hodgman (1961) and Cohen (1970). If $D_j$ is deposit per customer $μ=1, 2, \ldots, M$, at bank $j=1,2,\ldots,J$, the average balance-sheet ratio of earning assets, $A_j$, to average deposits, $D_j$, it can support is: $D_j(A/D_j)$, earning a revenue of $θD_j(A/D_j)$ and hence, a profit $Π_j= θD_j(A/D_j)−c_{\mu j}$, where $θ$ is the return on assets, net of asset costs. Under perfect competition, $Π_j=0$ and $D_{\mu \min}=(1/θ)(A/A_j)c_{\mu j}$, where $c_{\mu j}$ is the average deposit service costs per customer, and where the superscript “min” indicates that this is the minimum supply of deposit demands per customer. The total supply by bank $j$ is: $Σ_{μ}D_{μ \min}=D_{\min}$, and the total supply by all banks is: $(1/θ)Σ_{j}(D/A_j)c_{μ j}=(1/θ)(D/A)c$, so that $D_{\min}=(1/θ)(D/A)c$, given identical customers and identical banks.

Relating next the continuum of market structures with Cournot-type interaction in a Klein-Monti-Freixas and Rochet fashion (Klein 1971, Monti 1972, Freixas and Rochet 2008), the following general relationship between supply of demand deposits and market structure obtains:

$$D^s = \frac{J}{(1 + f)} \frac{1}{θ} \frac{D}{A} c,$$

where $s$ denotes supply, and $\lim_{f→0}D^s=D_{\min}$. This expression gives bank supply of demand deposits conditional on industry structure and on demand deposits already in the system. Letting the demand function for demand deposits be $D^d = ηT^a θ^γ$, with $η$, $a$, and $γ$ being constants, and $T$ being the appropriate demand constraint, equilibrium implies that:

$$ηT^a θ^γ = \frac{J}{(1 + f)} \frac{1}{θ} \frac{D}{A} c,$$

which equality yields that:

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A bank continues operating as long as $\theta \geq 0$. Letting $M$ and $h$ be the money stock and asset multiplier, respectively, $D = M/h$ and $A = M(1-h)/h = D(1-h)$, (3) becomes:

$$\theta = \left[ \frac{Jc}{(1+j)\eta T^a(1-h)} \right]^{1+\gamma}$$

which, when solved for $J$, gives:

$$J = \frac{\eta T^a(1-h)\theta^{1+\gamma}}{c - \eta T^a(1-h)\theta^{1+\gamma}} = \frac{D^d(1-h)\theta}{c - D^d(1-h)\theta}$$

$J$ can be positive if the denominator of (4) is positive, or the same, if:

$$\frac{c}{\theta} \geq D^d(1-h);$$

that is, if, from (1), the supply of deposits is at least:

$$D^s = \frac{J}{(1+j)} D^d(1-h) \frac{D}{A},$$

in which case, (2) gives:

$$D = \frac{D^d(1+j)A\theta}{Jc},$$

producing when solved for $J$, (4).

The positive sign of the derivative:

$$\frac{\partial J}{\partial \theta} = \frac{(1+\gamma)\eta T^a(1-h)\theta^\gamma c}{[c - \eta T^a(1-h)\theta^{1+\gamma}]^2} > 0$$

(5)

captures the fact that the bank number increases with the net return on assets; and so does with the discrepancy $(1-h)$:

$$\frac{\partial J}{\partial (1-h)} = \frac{\eta T^a \theta^{1+\gamma} c}{[c - \eta T^a(1-h)\theta^{1+\gamma}]^2} > 0.$$
To get a glimpse of the mechanics of such changes in connection with the public debt below, let us solve (7) for $i$:

$$i = \frac{1}{\varepsilon} [\theta - i_L(1 - \varepsilon) + i_D],$$

which in view of (3') becomes:

$$i = \frac{1}{\varepsilon} \left( \left[ \frac{f c}{(1 + j)\eta T^a (1 - h)} \right]^{1+\gamma} - i_L(1 - \varepsilon) + i_D \right) \tag{8}$$

The bond rate is directly related with the deposit rate, $\partial i/\partial i_D = 1/\varepsilon > 1$, and inversely related with the loan rate, $\partial i/\partial i_L = -(1 - \varepsilon)/\varepsilon < 0$. It is also directly related with $J$ and $h$:

$$\frac{\partial i}{\partial j} = \frac{1}{\varepsilon (1 + \gamma) j (1 + j)} \left[ \frac{j}{(1 + j) n (1 - h) T^a} \right]^{1+\gamma} > 0 \tag{9}$$

$$\frac{\partial i}{\partial h} = \frac{1}{\varepsilon (1 + \gamma) (1 - h)} \left[ \frac{j}{1 + j n (1 - h) T^a} \right]^{1+\gamma} > 0 \tag{10}$$

Let us now introduce into the discussion the debt-to-GDP ratio, $b$, as for example $b$ is contemplated by European Commission studies (see e.g. Berti, de Castro and Salto 2013):

$$b = b_{-1} \frac{1 + i}{1 + g} = pbal \tag{11}$$

where the subscript “-1” designates one-period lag, $i$ is the nominal interest rate, $g$ is the growth rate of nominal GDP, and $pbal$ denotes the primary budget balance. In view of (8), (11) becomes:

$$\frac{b}{b_{-1}} = \frac{1 + \frac{1}{\varepsilon} \left[ \left[ \frac{f c}{(1 + j)\eta T^a (1 - h)} \right]^{1+\gamma} - i_L(1 - \varepsilon) + i_D \right]}{1 + g} = \frac{pbal}{b_{-1}} \tag{12}$$

and hence, the derivatives of $b/b_{-1}$ with respect to $i_L$, $i_D$, $J$, and $h$, are $1/(1+g)$ times the corresponding derivatives of $i$. This explains the sign of the derivatives regardless any fiscal consolidation effort; the accumulation of debt raises $J$ by increasing $\varepsilon$ and $(i - i_D)$. The government competes with the private sector for bank loans by raising $i$ as a means of gaining preferential status given $i_L$ and $i_D$. This encourages the entry of new banks into the system to profit from deals with the government. A higher in turn number of banks can accommodate a higher volume of government borrowing from them; and, a higher volume of government borrowing encourages further entry of new banks into the system to profit by holding it, and so forth.

To accommodate the government its increasing dependence on bank borrowing, it also raises the reserve ratio, reducing the asset multiplier altogether, thus checking the expansion process. The private sector, which bids up $i_L$ to compete for bank funds, contains the increases in $i$ and $J$, too. A further deterrent factor comes from the fact that governments borrow also from the financial markets in which according e.g. to Diamond (1997) make up for the suboptimal presence of private investors in it, fostering efficiency in these markets, at the expense thereby of bank sector size: In any case, the operation of the Modigliani-Miller theorem is hampered by the presence of the government. Another factor that might mitigate its violation stems from Besanko and Kanatas’s (1993) observation that once a bank lends, it has an incentive to monitor the borrower, who however can now take recourse to the financial markets. And, a borrowing government might be

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thought of ideally as exhibiting no different behavior. Nevertheless, when government borrowing is extensive, governments are usually found to be intervening in the bank sector directly or indirectly, (see e.g. Zheng, et al. 2013), and in our case here, in order to ensure further government borrowing.

2.3 The Dynamics and Policy Considerations

With or without such intervention, the whole process is an unstable one as the signs of the following second-order derivatives:

\[
\frac{\partial^2 i}{\partial f^2} = \frac{yc}{\epsilon(1 + \gamma)^2[2(1 + f)\eta T^a(1 - h)]^{1+\gamma}} > 0,
\]

\[
\frac{\partial^2 i}{\partial h^2} = -\frac{\gamma}{\epsilon(1 + \gamma)^2[(1 + f)\eta T^a2(1 - h)]^{1+\gamma}} < 0,
\]

in conjunction with the fact that: \(\partial i/\partial i_L = (1-\epsilon)/\epsilon\), and hence, that:

\[
\frac{\partial i}{\partial f} = \frac{1-\epsilon}{\epsilon} \frac{\partial i_L}{\partial f}, \quad \frac{\partial^2 i}{\partial f^2} = \frac{1-\epsilon}{\epsilon} \frac{\partial^2 i_L}{\partial f^2},
\]

and \(\frac{\partial i}{\partial h} = \frac{1-\epsilon}{\epsilon} \frac{\partial i_L}{\partial h}, \quad \frac{\partial^2 i}{\partial h^2} = \frac{1-\epsilon}{\epsilon} \frac{\partial^2 i_L}{\partial h^2}\),

appear to suggest. Figure 1 illustrates the curves implied by these relationships and the consequent instability all the way up to \(h=1\), when (9) and (10) become equal to zero. The dynamics exhibit the properties of a stable limit cycle prompted by the mechanics described in the end of the last section: an increase in debt raises interest rates, encouraging subsequently bank entry, feeding back to a further increase in debt. Note that at \(h=1\), \(i\) and \(J\) obtain their maximum values, too. That is, the most a bank industry can become competitive is under such a reserve requirement ratio, \(\rho^*\), as to be rendering \(h=1\), in which case profits are zero, because there can still be lending but not bank money expansion. Under this traditional rather than “workable” notion of perfect bank competition, positive bank profits are rents from bank money expansion, rents known as commercial bank seigniorage, \(V\). Indeed, Baltensperger and Jordan’s (1997), for instance, definition of such seigniorage is, \(V=[i_L(1-\rho)-i_D]D\): Setting \(\rho=(i_L-i_D)/i_L\), one obtains that \(V=0\): this \(\rho\) is presumably the value of \(\rho^*\). Noting that: \((\partial i/\partial i_L)/(\partial i/\partial i_D)=\partial i_L/\partial i_D=(1-\epsilon)/(1-\epsilon)=1/(1-\epsilon)\), i.e. that the change in \(i_L\) exceeds that in \(i_D\) and hence, that the process leading to \(i_L^{max}\) leads also to \(i_D^{max}<i_L^{max}\), the spread \((i_L-i_D)\) continues being positive, and banks do continue earning profits under perfect competition. Yet, these profits are the normal ones, covering opportunity cost, and are distinct from commercial bank seigniorage and from the zero accounting profit.

Solving the commercial bank seigniorage equation for \(i_L\): \(i_L=(V+i_D D)/(1-\rho)D\), and inserting it in (6):

\[
i = \frac{1}{\epsilon} \left\{ \frac{Jc}{(1 + f)\eta T^a(1 - h)} \right\}^{1+\gamma} - \frac{V + i_D D}{(1 - \rho)D} (1 - \epsilon) + i_D
\]

(8')
Figure 1 Debt accumulation and bank numbers

The sign of the derivative:

\[ \frac{\partial i}{\partial V} = - \frac{1 - \varepsilon}{\varepsilon(1 - \rho)D} < 0, \]

ascertains even further the conclusions reached so far about bank industry structure and its interplay with public debt: \( i \) increases as \( V \) decreases. From a policy point of view, calls for a 100% reserve requirement should actually be calls for such a \( \rho \) that inhibits bank money expansion, which is achieved through \( \rho^*=(i_L-i_D)/i_L<1 \), if, of course, the target is to nullify commercial bank money creation and seigniorage. Also, from the viewpoint of a policy against (excessive) debt accumulation, it is clear that raising \( \rho \) and through this \( h \) too, will make it difficult for banks to hold the public debt: \( b \) obtains its maximum at \( h=1 \), having \( \rho \) reached its maximum value, \( \rho^* = \rho^{\text{max}} \), which from the last expression for \( \rho^* \), implies that \( i_L^{\text{max}} = i_D/(1-\rho^*) \) under the appropriate value of \( i_D \), too. From still another debt policy view: Would a government, wishing the continuation of the accumulation of debt, want to embrace the policy prescription of setting \( \rho=\rho^* \)? The answer is not only negative but that such a government would want in addition to check the process towards \( h=1 \) by intervening in the banking system directly as e.g. Zheng et al. (2013) maintain, worsening subsequently the debt problem.
Finally, if perfect bank competition is considered to be the optimal one, optimal will be the number of banks comprising a banking sector operating by covering only its opportunity cost. That is, it is a “relativistic” definition, since we have seen that normal only bank profit may be attained in a number of ways. A monopoly however bank would be suboptimal under any circumstances as follows: Solving (8) for \( J \), yields that:

\[
J = \frac{\eta T^a [\epsilon i + i_L (1 - \epsilon) - i_D]^{1+\gamma}}{c - \eta T^a [\epsilon i + i_L (1 - \epsilon) - i_D]^{1+\gamma}}
\]  

(13)

Assuming that \( b/b_i = 0 \) and a balanced primary budget in (12), and solving for \( J \), gives:

\[
J = \frac{\eta T^a [i_L (1 - \epsilon) - i_D - \epsilon]^{1+\gamma}}{c - \eta T^a [i_L (1 - \epsilon) - i_D - \epsilon]^{1+\gamma}}
\]  

(14)

One concludes from their ratio that:

\[
1 < \frac{\eta T^a [\epsilon i + i_L (1 - \epsilon) - i_D]^{1+\gamma}}{\eta T^a [i_L (1 - \epsilon) - i_D - \epsilon]^{1+\gamma}} = \frac{c - \eta T^a [\epsilon i + i_L (1 - \epsilon) - i_D]^{1+\gamma}}{c - \eta T^a [i_L (1 - \epsilon) - i_D - \epsilon]^{1+\gamma}}
\]  

(15)

The numerator in the left-hand fraction is greater than one; but, less so were Friedman’s rule, \( i_0 = 0 \), to be assumed. Consequently, under fiscal consolidation with \( pbal = 0 \), and hence, under declining \( i \), (given from (11) that \( d(b/b_i)/di = 1/(1 + g) > 0 \)), bank competition lessens but not to the point of ending up in one only bank monopolizing the bank sector. Anyway, the overall policy prescription emerging from these considerations regarding fiscal consolidation is the encouragement of a restructuring of the bank sector towards a smaller one in terms of number of banks, operating with normal only profit.

This policy prescription follows directly from our equations, but needs to be discussed further, because it hinges upon the intergenerational dimension of the public debt. Debt holding by the banks serves \textit{inter alia} as a risk-sharing device against intergenerational consumption risks associated with illiquidity shocks in a Diamond and Dybvig (1983) and Holmstrom and Tirole (1998) fashion. Banks simply provide insurance to individuals and firms against such shocks, because the competitive Arrow-Debreu insurance markets cannot do it efficiently. From this point of view, our policy prescription is one in the spirit of the proposition that Friedman’s rule generates efficiency when banks enjoy some monopolistic power, as contemplated above, and when combined with a proper discount window regardless bank industry structure (sees e.g. Matsuoka 2011). Our overall policy prescription should be complemented in compliance with the second part of this proposition, because setting \( i_L = (V + i_D + (1 - \rho) D)/\rho \) in (13) and (14), the left-hand fraction of (15) is replaced by the ratio:

\[
\frac{i_D^* (\rho^* - \epsilon)}{i_D^* (\rho^* - \epsilon) - \epsilon (1 - \rho^*)}
\]  

(15’)

which obtains when \( i = 0 \) and \( i_D^* = i_L (1 - \rho^*) \Rightarrow V = 0 \). (15’) holds regardless the size of \( V \), and becomes (15’) when \( V = 0 \). It is worth noting that under a full-reserve requirement, \( \rho^* = 1 \), (15’) becomes equal to one, that is, such a requirement would dictate monopoly banking, which according to (15) would be suboptimal. Nevertheless, a similar result may be obtained through other combinations of \( \rho^* \), \( i_D^* \), and \( \epsilon \). In general, (15’) is greater than or equal to unity, since \( 1 \geq \rho^* \geq \epsilon \geq 0 \Rightarrow 0 \Rightarrow 0 \Rightarrow 0 \Rightarrow i_D^* (\rho^* - \epsilon) \geq i_D^* (\rho^* - \epsilon) - \epsilon (1 - \rho^*) \). It is illustrated arithmetically by Table 1 in the Appendix, based on data from 35 developing economies.

Anyway, we just saw that once intergenerational considerations are introduced into the discussion, the standard-textbook notion of perfect competition becomes suboptimal relative to the notion of workable competition in so far as the optimal number of banks is concerned. The latter
notion does not preclude the presence of positive profit and such should be the profit if optimality is sought from the intergenerational point of view. Bank profits are simply rents from the issuance of the bank money needed to prevent disturbances in intergenerational transfers that would disrupt liquidity and the course of debt accumulation. Judging the dynamics between debt evolvement and bank numbers from the viewpoint of the textbook concept of perfect competition circumvents the intergenerational dimension of this interaction. And, this, in turn disregards the possibility of generation-specific bank-runs against which banks have to be insured by creating bank money out of profit.

3. Conclusion and Remarks

In any case, the optimal number of banks has been found to be related intimately with the course of debt accumulation; intimately, but not uniquely, because a given debt level may be consistent with different numbers of banks depending on the overall government policy. From the point of view of policy regarding banking with a view towards fiscal consolidation, a limited number of “systemic” banks, operating efficiently would be in line with workable competition. This “efficiently” touches upon the issue of the size distribution of banks, which our formal modeling assumes away; all banks are assumed to be identical. If the number of banks expands with debt accumulation, it means that originally there were a number of banks of certain size whose increasing debtholding prompts the entry into the industry of new banks to unbend the subsequent rationing of private loans and take advantage of bond rates that compete with lending rates. Are the new banks smaller or larger than the original ones?

If we assume that entry does not alter bank size and at the same time that the optimal would be a few systemic banks, it will be equivalent to claiming that entry weakens uniformly over the industry the economies of scale and scope needed to avoid intergenerational liquidity disturbances. At the other end, if we maintain that entry does alter the size distribution of banks, the following two are the cases that should be contemplated. Either that originally there was either a competitive fringe in which the smallness of bank size was prohibitive of realizing fully the improved scale prospects, and new bigger banks had to come and fill the gap. Or, that originally there were a few banks which were refraining from taking advantage of these prospects just because profit maximization under monopoly power dictates so, leaving thereby room for the entrance of new smaller firms to come and benefit from covering the portion of the market left over by the large banks.

It is clear therefore that the size distribution of banks is an empirical case-by-case matter; the fact remains that fiscal consolidation presupposes the restructuring of the industry towards a few healthy number of banks, which certainly cannot be the too-big-to-fail ones. Fiscal consolidation per se presupposes the fewness of banks, since it is based on austerity, which in turn affects adversely banking business. The whole policy matter towards the bank sector is a political economy one; a matter of bank regulation and supervision and of contract enforceability given that “political economy variables not only influence market structure directly but also affect the relation between efficiency and structure” (González 2009, p. 737) Opportunistic governments which are typically driven by partisan monetary policy (see e.g. Drazen 2001) might upset considerably the industry, which is simply a “must not do” (see, e.g Llewellyn 2006).

Acknowledgement: I am grateful to two anonymous reviewers for their useful comments and suggestions. Any remaining errors are my own.
References


**Appendix**

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~ 50 ~
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<tr>
<th>Country</th>
<th>Debtholding %</th>
<th>Reserve Req %</th>
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<td>Panama</td>
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Calculations based on data from the World Bank:
http://data.worldbank.org/indicator/FR.INR.LNDP/countries,
http://data.worldbank.org/indicator/FR.INR.LEND/countries,

\( \rho^* \) is calculated as if the current country interest rates were the ones consistent with \( V=0, i=0, \) and \( \rho=1; \) the optimal number of banks is thereby one; and the increase in \( \varepsilon \) indicates the increase in the percentage of bonds held in this bank’s portfolio. Debtholding operates like a more than 100% reserve requirement for the bank. For example, given the value of \( \rho^* \) for Algeria, the number 1.285714 indicates that 50% of the bank’s portfolio in the form of bonds is equivalent to having it under a reserve ratio, \( \rho, \) equal to 128.5714%; or the same, 0.285714 more banks would be needed like the one operating under \( V=0, i=0, \) and \( \rho=1, \) i.e. like the monopoly bank, to make possible such a debtholding under \( V=0, i=0, \) and \( \rho=1. \)