Profitability, Growth, and Different Flow Ratio Concepts: Implications for Failing Firms

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Abstract: The objective is to develop a mathematical model of the firm to show the relationship between profitability, growth, and financial flow concepts especially under conditions for failing firms. It is assumed that revenue flows are generated by periodic expenditures growing at a steady rate. These revenue flows are described in terms of profitability (internal rate of return), growth, and time lag between invested expenditure and generated revenue flow. Three kinds of financial flow concepts are drawn: revenue-expenditure flow (quick flow), revenue-expense flow (earnings), and cash flow. Earnings are drawn for three depreciation theories: proportional, rate of return, and compound interest depreciation. For each concept, flow ratios are drawn and compared with each other. Theoretical results are illustrated by empirical data from Finnish non-failing and failing firms.

JEL Classifications: G32, M41, G33, C65

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1. Introduction

Financial ratios are widely used by stakeholders to analyse the financial performance of the firm and to predict its future success or failure. Financial ratios reflect key relationships between financial variables and provide basic guidelines for financial planning and analysis. In spite of observed decline in value relevance, these ratios include important information for all stakeholders (Balachandran & Mohanram, 2011). They are principally used in two different ways (Whittington, 1980). First, ratios are used to measure the ratio of a firm to compare it with a standard (normative use). Second, they are used in estimating empirical relationships, usually for prediction (positive use). There are two principal reasons for using ratios (Barnes, 1987: 450). First, they are used to control for the size effect on the financial variables being analyzed (control of size). Second, they are used to control for industry-wide factors (control of industry effect). In summary, financial ratios are important tools for business analyses and predictions.

Financial ratio analysis is suffering from lack of theoretical research. Horrigan (1968: 294) states that the most striking aspect of ratio analysis is the absence of an explicit theoretical structure. Thus, the subject of ratio analysis is replete with untested assertions about which ratios should be used and what their proper levels should be. The expected relationships of the various ratios with quantification have not generally been formulated (Horrigan, 1968: 294). Barnes (1987: 457) states that even if we may be much nearer to the theory of financial ratios to which Horrigan referred, there are several aspects in which there has been little advance. For example, financial ratios are rarely linked to theories of economic and financial behaviour. This empiricism has been critical...
especially in financial failure studies (Balcaen & Ooghe, 2006: 79-81; Lensberg, Eilifsen & McKee, 2006: 678-679). The lack of a theoretical basis has caused serious drawbacks as a form of sample specific results (Balcaen & Ooghe, 2006: 80). Thus, there is a strong call for theoretical research of financial ratio analysis.

The objective of this study is to develop a theoretical basis for financial flow ratios. It aims to explain how different financial factors affect flow ratios based on different flow conventions, especially in the context of failing firms. Flow ratios are important, since they include distinct information dependent on accounting conventions for time of recognition (Barlev & Livnat, 1990; Hung, 2001). For example, cash flow information may not be captured by any accrual ratios (Gombola & Ketz, 1983: 113; Casey & Bartczak, 1985; Aziz, Emanuel & Lawson, 1988; Akbar, Shah & Stark, 2011). However, failure research tends to report that cash flow information may not remarkably add value to accrual ratios (Gombola, Haskins, Ketz & Williams, 1987; Laitinen, 1994; Sharma, 2001; Joseph & Lipka, 2006). Sharma (2001) argues that it can be the limitations of the research that can be the reason for this and not necessarily its potential information content. Thus, empirical evidence on the relevance of flow ratios is mixed calling for theoretical research (Sharma & Iselin, 2000; 2003a; 2003b). This study aims to be a response to this call.

Flow ratios measure the sufficiency of revenue finance based on different accounting conventions. In this study, three kinds of flow statements are theoretically considered (cf. Staubus, 1966): 1) revenue-expenditure flow (quick flow), 2) revenue-expense flow (accrual flow or earnings), and 3) cash revenue-cash expenditure flow (cash flow). The quick flow reflects revenue finance in terms of the difference between periodic revenue and expenditure (expenditure or quick margin). This quick margin does not record any operating financial transactions such as collection of accounts receivable and payment of trade debt (accounts payable). However, if there is a deficiency in revenue finance, the firm can increase finance by collecting accounts receivable efficiently and postponing payments of accounts payable. In this way, the firm can show a difference between cash revenue and cash expenditure (cash margin) higher than the quick margin. The accrual flow (earnings) is important in determining the accounting (book) profit (margin) as a difference between periodic revenue and matched expense. The difference between cash flow and earnings (accruals) is still a controversial issue especially in analysing failing firms (Sharma, 2001; Al-Attar, Hussain & Zuo, 2008; Akbar, Shah & Stark, 2011; Jiang & Stark, 2011; Badertscher, Collins & Lys, 2012).

The three flow statements are here described by a dynamic model based on a revenue finance approach. In this framework, the firm is assumed to consist of successive periodic expenditure flows which generate separate flows of revenue (Solomon, 1966; Fisher & McGowan, 1983; Laitinen, 1991; 1997; 2006; Ely & Miller, 2001; Stark, 2004; Said, HassabElNabay & Nowlin, 2008). This characteristic is called an assumption of identical investment projects. The revenue generating process is described by the internal rate of return (IRR), the rate of growth, and the time lag between expenditure and revenue flows. IRR represents the true profitability of the firm while growth is a factor affecting the need for outside finance. The time lag refers to the technology capturing thus the industry effect (Smith & Liou, 2007; Hossari, 2009). Earnings are drawn for three depreciation theories: proportional, rate of return, and compound interest depreciation. Profit-sharing items (taxes, interest payments, and dividends) are calculated based on simplified assumptions. Gross margins and net margins (after profit-sharing items) are extracted for each flow concept and compared with each other. The findings are interpreted especially for failing firms. This kind analysis is important because there is little research on the dynamics of failing firms (Balcaen & Ooghe, 2006; Joseph & Lipka, 2006; Ooghe & De Prijcker, 2008; Fitzpatrick & Ogden, 2011).

In summary, the purpose is to draw theoretical results which would help us in understanding the role of different flow ratios in financial ratio analysis, especially in context of failing firms. The structure of the paper is as follows. The background and purpose of the study are briefly explained
in this introductory section. In the second section, the theoretical model is introduced and the basic concepts are drawn. The gross margin and net margin ratios based on earnings, quick flow, and cash flows are theoretically analysed and compared with each other in the third section. In the fourth section, raw empirical figures from Finnish non-failing and failing firms are used to illustrate theoretical results. Emphasis is set on analysing flow ratios in different types of failing firms. Finally, the fifth section summarizes the findings of the study.

### 2. Mathematical Model of Financial Flows

#### 2.1 Revenue Finance

##### 2.1.1 Model

The present model is a version of approaches based identical investment projects originally developed to show the relation between the return on investment ratio and \( IRR \) (Solomon, 1966; Fisher & McGowan, 1983; Laitinen, 1991; 1997; 2006; Stark, 2004). It is assumed that each periodic total expenditure \( E(t) \) generates proportionally an identical flow of revenue converging geometrically at a constant rate \( q \) towards infinity. Each unit of expenditure will generate \( M \) units of revenue. \( IRR \) \((r)\) of expenditure \( E(t) \) is defined as the identity:

\[
E(t) = E(t)M(1-q)\sum_{i=0}^{\infty}q^i(1+r)^{-i} \Rightarrow M = \frac{1+r-q}{(1+r)(1-q)} \quad (1)
\]

which holds for \( q/(1+r) < 1 \) and \( q < 1 \). Here, \( q \) reflects the lag between expenditure and revenue the average time lag being \( K=\frac{q}{1-q} \).

It is assumed that \( E(t) \) will grow at a constant rate \( g \) over time. \( R(t) \) realized in period \( t \) can be expressed as the sum of revenue contributions generated by past and current expenditure as follows:

\[
R(t) = E(t)M(1-q)\sum_{i=0}^{\infty}q^i(1+g)^{-i} \Rightarrow R(t) = E(t)\frac{(1+r-q)(1+g)}{(1+r)(1+g - q)} \Rightarrow \\
\frac{R(t)}{E(t)} = \frac{(1+r-q)(1+g)}{(1+r)(1+g - q)} = F(t) = F
\quad (2)
\]

for \( q/(1+g) < 1 \) and \( q < 1 \). \( F \) is the revenue-expenditure ratio which refers to the rate of revenue finance. \( F \) is symmetric with respect to \( r \) and \( g \). Thus, \( F \) equals unity when \( r = g \) and exceeds unity when \( r > g \).

##### 2.1.2 Discussion

The model shows that the rate of revenue finance is the higher, the higher is \( IRR \) and the lower is the rate of growth. The results in (1) and (2) are valid also for negative \( IRR \) and for negative growth. Therefore, the model is useful also for analyzing failing firms. For a failing firm, \( IRR \) is typically low and so is the rate of revenue finance. If this kind of firm grows rapidly, it may suffer from a serious lack of revenue finance probably leading to failure. If both \( IRR \) and \( g \) are negative so that \( IRR < g \), the rate of revenue finance is extremely low. If the firm tends to grow at the rate equal to \( IRR \), it can finance its expenditure by revenue. This growth path can be called the golden rule (path) of revenue finance.

The effect of the time lag on revenue finance depends on the relation between growth and \( IRR \). If \( IRR \) exceeds the rate of revenue finance, the rate of revenue finance is increasing in the lag. However, if the rate growth exceeds \( IRR \), the relation is reversed. In practice, the time lag refers to the industry (technology) effect recognized in financial statement analysis (Smith & Liou, 2007). Thus, in fast-growing industries \((g > IRR)\), a technology with a long time lag tends to decrease the rate of
2.2 Assets and Expenses

2.2.1 Rate of return depreciation method

In this study, three different depreciation or valuation theories are applied to draw earnings (cf. Laitinen, 2006). First, the rate of return or realization method of depreciation is associated with the revenue flow generated by an investment project (Saario, 1961; Bierman, 1961). It suggests that it is necessary to view the purchase of an asset as an acquisition of a series of revenue generating services, rather than as a purchase of a physical unit. Therefore, expenses are determined by the present value of realized revenue, discounted by the internal rate of return to the moment of acquisition. This present value is regarded as the expense of the revenue. The adoption of the realization theory leads to an accelerated depreciation pattern when $\text{IRR} > 0$. This theory assumes that expenses are determined as follows:

$$C(t) = E(t)M(1-q)\sum_{i=0}^{\infty} q^i[(1+g)(1+r)]^{-i} \Rightarrow C(t) = E(t)\frac{(1+r-q)(1+g)}{(1+r)(1+g)-q}$$  \hspace{1cm} (3)$$

where $C(t)$ is the depreciation (expenses) in the period $t$.

The accounting identity shows that periodic expenses can be determined as the difference between the periodic expenditures $E(t)$ and the periodic change in assets $A(t)$ in the following way:

$$C(t) = E(t) - A(t) + A(t-1) \Rightarrow A(t) = [E(t) - P(t)]\frac{1+g}{g}$$ \hspace{1cm} (4)$$

for $g \neq 0$. Then, inserting (3) in (4) gives the assets:

$$A(t) = E(t)\frac{q(1+g)}{(1+r)(1+g)-q}$$ \hspace{1cm} (5)$$

which is decreasing in $r$ and $g$ but increasing in $q$.

2.2.2 Proportional depreciation method

Second, the proportional depreciation theory leading to a neutral depreciation pattern is used. This theory follows the philosophy adopted in accounting practice. In the annual closing of the books, periodic expenditures are classified into two categories, expenses (expired expenditures) and unexpired expenditures. The unexpired expenditures have not yet generated realized revenues and they will be added to the assets of the firm. These assets are got by summing the unexpired proportions of expenditures up to infinity as follows:

$$A(t) = \sum_{j=0}^{\infty} E(0)(1+g)^{t-j} \sum_{i=j+1}^{\infty} (1-q)q^i = E(t)\frac{q(1+g)}{1+g-q}$$ \hspace{1cm} (6)$$

which does not depend on $\text{IRR}$ but is decreasing in $g$ and increasing in $q$.

The accounting identity makes it possible to calculate total expense $C(t)$ from (6):

$$C(t) = E(t) - A(t) + A(t-1) = E(t) - A(t)\frac{g}{1+g} = E(t)\frac{(1+g)(1-q)}{1+g-q}$$ \hspace{1cm} (7)$$

which shows that the rate of expense is increasing in $g$ but decreasing in $q$. Equations (6) and (7) are independent of $\text{IRR}$. 

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2.2.3 Compound interest depreciation method

Third, the last asset valuation theory used is the compound interest or annuity depreciation originally presented by Ladelle in 1890 (Wright, 1967). It is based on the economic wealth assumption according to which the financial position of the firm is related to its future revenues. Consequently, the assets of the firm are determined as the present value of future revenues generated by past and present expenditures, and that will be realized in the future. In this present value, \( IRR \) is used as the rate of discount leading to a decelerated depreciation pattern when \( IRR > 0 \). This leads to the following result:

\[
A(t) = \sum_{j=0}^{\infty} E(0)(1 + g)^{t-j} \sum_{i=j+1}^{\infty} (1-q)^i (1+r)^{j-i} = E(t) \frac{q(1+g)}{(1+r)(1+g-q)} \quad (8)
\]

Substituting (8) into (4) the periodic expenses are obtained as:

\[
C(t) = E(t) \frac{(1+g-q)(1+r)-qg}{(1+g-q)(1+r)} \quad (9)
\]

showing that \( C(t) \) is decreasing in \( q \) but increasing in \( r \) and \( g \).

2.2.4 Discussion

The three depreciation theories lead to different expenses and assets when \( IRR \) and \( g \) are non-zero. If \( g = 0 \), periodic expenditure equals periodic depreciation for all depreciation theories. However, these theories lead to different book assets and thus to different return on asset ratios. If \( IRR \) equals zero, all three theories give the same expression for expenses and assets as the proportional depreciation that is independent of \( IRR \). The lower \( IRR \), the weaker is the effect of the depreciation method on expenses and assets. Therefore, in failing firms with a low \( IRR \), depreciations and assets are relatively insensitive to the depreciation method. For \( IRR > 0 \), the use of the realization depreciation method gives the highest (lowest) periodic depreciation (assets) whereas the proportional depreciation method leads to the lowest (highest) depreciation (assets). For \( IRR < 0 \), the order of depreciations is however reversed.

Thus, in a failing firm with negative \( IRR \), the accelerated depreciation method leads to the lowest depreciation, and the neutral method to the highest. This is as expected, since the rate of discount for future revenues is negative. The only difference between the annuity depreciation and the proportional depreciation is that the former assumes discounting but the latter does not. The time lag is essential when analyzing the effect of depreciations. If the time lag is zero so that the moment of acquisition and the moment of disposal are equal (all revenues are generated instantly), there is no need for depreciations.

2.3 Profit-sharing Items

2.3.1 Taxes, interest payments, and dividends

It is assumed the debt \( B(t) \) and the equity \( S(t) \) at the end of period \( t \) can be expressed as a function of total assets \( A(t) \) in the following way:

\[
B(t) = bA(t) \\
S(t) = (1-b)A(t) \quad (10)
\]

where \( b \) is the constant book debt-to-assets ratio (DAR).

Interest payments \( l(t) \) and dividends \( D(t) \) will be determined on the basis of the debt and the equity on the beginning-of-year basis as follows:
\[ I(t) = ibA(t - 1) \]
\[ D(t) = d(1-b)A(t - 1) \]

where \( i \) and \( d \) are constant rates of interest and dividend, respectively.

Income tax \( T(t) \) is based on the taxable profit that is defined as the difference between the profit margin \( PM(t) = R(t) - C(t) \) and interest payments \( I(t) \) as follows:

\[
T(t) = \begin{cases} 
  f( PM(t) - I(i)) & \text{for } PM(t) - I(t) \geq 0 \\
  0 & \text{for } PM(t) - I(t) \leq 0
\end{cases}
\]

where \( f \) is the constant income tax rate. The weighted average cost of capital after tax (WACC) can be defined as:

\[ w = (1-f)ib + d(1-b) \]

that is constant.

2.3.2 Discussion

The three depreciation theories lead to different profit-sharing terms when DAR is fixed. For an identical \( b \), the proportional depreciation leads to the highest interest payments \( I(t) \) and dividends \( D(t) \) while the realization depreciation gives the lowest ones, when \( r > 0 \) and \( g > 0 \). If \( r < 0 \), then reversed results are got. If \( r = 0 \), all depreciations give identical results. However, the effect of depreciation theory on taxes \( T(t) \) is more complicated. For the annuity depreciation, \( T(t) \) is positive when \( r > ib \). The proportional depreciation gives higher \( T(t) \) than the annuity depreciation when \( r(g-ib) > 0 \). If \( g = ib \), the results are identical. For this depreciation, the positivity condition for \( T(t) \) is thus \( r > ib/(1+g-ib) \). For the realization depreciation, \( T(t) > 0 \) when \( rF > ib \). For \( r > 0 \), this depreciation gives higher \( T(t) \) than the annuity depreciation when \( F > 1 \) or when \( r > g \). It also gives higher \( T(t) \) than the proportional depreciation when \( F > 1+g-ib \). When \( r = 0 \), all depreciations give a non-positive profit so that in that case \( T(t) \) is equal to zero.

3. Quick Flow, Earnings, and Cash Flow Margins

3.1 Gross Margins

3.1.1 Gross quick margin

Equation (2) for \( F \) can be used to calculate directly the gross quick margin \( GQM \) as a ratio of \( R(t) - E(t) \) to the revenue \( R(t) \) as:

\[
\frac{R(t) - E(t)}{R(t)} = GQM = 1 - \frac{q(r-g)}{F(1+r-q)(1+g)}
\]

which shows that \( GQM \) is increasing in \( r \) but decreasing in \( g \). The effect of \( q \) depends on the relationship between \( r \) and \( g \). For \( r > g \), \( GQM \) is positive and increasing in \( q \) but it is negative and decreasing when \( r < g \).

3.1.2 Gross profit margin

The gross profit margin \( GPM \) or earnings before interest and taxes (EBIT) is expressed as the ratio of \( R(t) - C(t) \) to \( R(t) \) as follows:

\[ GPM = \frac{[R(t)-C(t)]}{R(t)} \]
Thus, Equation (2) together with (4), (7), and (9) can be used to draw the gross profit margin. The expressions of (15) for the three depreciation theories are presented in Table 1. For each depreciation theory, gross profit margins are zero when \( r = 0 \) and positive when \( r > 0 \). These variables are increasing in \( r \). However, the effect of \( g \) depends on the sign of \( r \). The variables are decreasing in \( g \) when \( r > 0 \) and increasing when \( r < 0 \), with one exception. For the proportional depreciation, \( GPM \) is independent of \( g \). For \( g = 0 \), all depreciations give identical expressions. For \( g > 0 \), the proportional depreciation gives the highest gross profit margin ratio while the lowest ratio is given by the realization depreciation. For \( g < 0 \), the effect of depreciation methods is however reversed.

### Table 1. Gross profit margin (GPM) for different depreciation theories

<table>
<thead>
<tr>
<th>A. Realization depreciation</th>
<th>B. Proportional depreciation</th>
<th>C. Annuity depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>( GPM = \frac{rq}{(1+r)(1+g) - q} )</td>
<td>( GPM = \frac{rq}{1 + r - q} )</td>
<td>( GPM = \frac{rq}{(1 + g)(1 + r - q)} )</td>
</tr>
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</table>

#### 3.1.3 Gross cash margin

The gross margin based on the cash flow is derived assuming that the cash revenue \( CR(t) \) and the cash expenditure \( CE(t) \) are expressed as a function of accrual concepts:

\[
CR(t) = R(t) + uR(t) \frac{1}{1 + g} - uR(t) = R(t) \frac{1 + g(1-u)}{1 + g} \\
CE(t) = E(t) + pE(t) \frac{1}{1 + g} - pE(t) = E(t) \frac{1 + g(1-p)}{1 + g}
\]

where \( u \) is the proportion of revenue not paid in cash and \( p \) is the proportion of expenditure not paid in cash. \( u \) is called here the proportion of accounts receivable in revenue and \( p \) the proportion of accounts payable in expenditure.

The gross cash margin ratio \( GCM \) can be expressed as the ratio of \( CR(t) - CE(t) \) to \( CR(t) \) so that using (2) and (16) we get:

\[
GCM = \frac{CR(t) - CE(t)}{CR(t)} = 1 - \left[ \frac{1 + g(1-p)}{1 + g(1-u)} \right] \frac{1}{F} = 1 - \left[ \frac{1 + g(1-p)}{1 + g(1-u)} \right] \frac{(1+r)(1+g-q)}{(1+r-q)(1+g)} \tag{17}
\]

If \( g = 0 \), then \( GCM = GQM \) and in that case \( GCM > 0 \) when \( r > 0 \). If \( u = p \), then \( GCM \) is positive when \( r > g \) as \( F \). If \( u > p \), then \( GCM \) is increasing in \( r \). It is also increasing in \( q \) when \( r > g \) but decreasing when \( r < g \). The parameters \( p \) and \( u \) make the effect of \( g \) on (17) complicated. \( GCM \) is decreasing in \( g \) for typical values of parameters.

#### 3.1.4 Discussion

The gross flow margins based on different accounting conventions are sensitive to different factors. The gross quick margin reflects the difference between \( IRR \) and \( g \). In a golden path when \( IRR = g \), the margin is zero. In that case, expenditures are equal to revenues. If \( IRR > g \), the margin is positive and negative in the opposite case. The time lag \( q \) also affects the margin. This effect depends on the relation between \( IRR \) and \( g \), that is, on the sign of the margin. If the margin is positive, higher \( q \) tends to show higher margins. If it is negative, it shows lower margins. However, the sensitivity of the ratio shows that low margins in failing firms are mainly a consequence of two possible situations: 1) \( IRR \) is low (in comparison to \( g \)) or 2) \( g \) is high (in comparison to \( IRR \)). The latter situation means that also a profitable firm can suffer from lack of revenue finance if \( g \) is very low.
high in comparison to $IRR$. If $IRR$ is however low and $IRR < g$ (as in situation 1), financial distress can be very serious and lead to failure. In the latter case, it is more difficult to get external finance to cover the deficiency in revenue finance.

The gross profit margin based on $EBIT$ is sensitive to $IRR$ supporting its validity in profitability measurement. This margin is positive when $IRR > 0$. For a failing firm with $IRR < 0$, the ratio is negative irrespective of the depreciation theory. The profit margin is not sensitive to the rate of growth. However, it is strongly affected by $q$ which together with $IRR$ determines the level of the ratio. This means that firms with longer lag can show higher margins than similar firms with shorter lag. This (industry) effect obviously weakens the validity of the ratio. The three depreciation theories lead to differences in ratios which weakens the reliability of the margin. However, the differences are sensitive to growth. If $g = 0$, all theories give identical ratios. Thus, in a zero-growth firm the depreciation method is not relevant from this perspective. In fast growing firms, the effect of depreciation is emphasized. In this situation, the proportional depreciation leads to the highest profit margin while the realization depreciation gives the lowest one. However, for a failing firm with $g < 0$, the effect of depreciation is reversed.

For the cash flow, the results are strongly affected by the growth of the firm. If $g = 0$ zero, the gross cash margin equals the gross quick margin. In this kind of zero-growth firm, the cash margin is positive when $IRR > 0$. For a firm with $g > 0$, the minimum $IRR$ to reach positive gross cash margin exceeds $g$ when the rate of accounts receivable is higher than the rate of accounts payable. When the rate of accounts payable is higher, this minimum $IRR$ is less than $g$. If the rates are equal, a positive gross cash margin is resulted when $IRR > g$. The effect of $q$ depends on the relation between $IRR$ and $g$. If $IRR > g$, firms with longer lag tend to show higher cash flows, and vice versa. In most situations, cash flows suffer from fast growth. For a failing firm with low $IRR$, the cash flow margin is typically very low. The firm has five ways to increase the margin. It can increase $IRR$, decrease $g$, speed up revenue generation for lower $q$, collect accounts receivable more efficiently, or postpone payment of accounts payable. In reorganizing processes, these activities are often made in this order, to avoid failure.

### 3.2 Net Margins

3.2.1 Net quick margin

The net quick margin $NQM$ is defined as:

$$NQM = \frac{R(t) - E(t) - I(t) - D(t) - T(t)}{R(t)} = \frac{R(t) - E(t) - W(t)}{R(t)} \tag{18}$$

where $W(t)$ refers to the profit-sharing items in (11) and (12). The expressions for $NQM$ are presented in Table 2. For the annuity depreciation, $NQM$ is positive when $r > (g+w)/(1-f)$. For the proportional depreciation the positivity condition is correspondingly $r > (g+w)/(1-f(1+g)-w)$. For the realization depreciation, $NQM$ is implicitly positive when $r > (g+w)/(1-F)$ where $F$ is a function of $r$. When $F = 1$ or $r = g$, the condition is the same as for the annuity depreciation. However, in that case $NQM$ cannot be positive for $W(t) > 0$, since $GQM = 0$. Therefore, $r > g$ or $F > 1$ and the requirement for $r$ is higher for the realization depreciation.

Equation (18) shows that $NQM$ depends on the depreciation theory only through $W(t)$. The higher $W(t)$, the lower is the margin. If $r = 0$, all the theories give the same result for $NQM$. $W(t)$ derived for the proportional depreciation exceeds that for other deprecations if $r > 0$ when $g > -w/f$. For this same condition, the annuity depreciation leads to higher $W(t)$ than the realization depreciation. Therefore, if $r > 0$ and $g > -w/f$, the realization depreciation gives the highest $NQM$ while the proportional depreciation gives the lowest ones. For $r < 0$, the order of the depreciation theories is reversed.
Table 2. Net quick margin (NQM) for different depreciation theories

A. Realization depreciation

\[
NQM = \frac{q\left[r(1-fF) - g - w\right]}{(1+r)(1+g) - qF} = \frac{q\left[r(1-fF) - g - w\right](1+g-q)(1+r)}{(1+r)(1+g) - qF(1+r-q)(1+g)}
\]

B. Proportional depreciation

\[
NQM = \frac{q\left[r(1-f(1+g) - w) - g - w\right]}{(1+g)(1+r-q)}
\]

C. Annuity depreciation

\[
NQM = \frac{q\left[r(1-f) - g - w\right]}{(1+g)(1+r-q)}
\]

3.2.2 Net profit margin (retained earnings)

For earnings, the net profit margin NPM is calculated as:

\[
NPM = \frac{R(t) - C(t) - I(t) - D(t) - T(t)}{R(t)} = \frac{R(t) - C(t) - W(t)}{R(t)}
\]

where NPM thus refers to retained earnings in a relation to revenue.

The expressions for NPM are presented in Table 3. For all depreciations, NPM is sensitive to \( q \). The proportional depreciation leads to higher NPM than the annuity one if \( r \) is positive and \( g > w/(1-f) \). In general, NPM is positive when \( ROI(1-f)-w > 0 \) which means that \( ROI \) after taxes exceeds WACC. For the annuity depreciation \( ROI = r \) so that \( NPM > 0 \) simply when \( r(1-f) > w \) or \( r > w/(1-f) \).

For the realization depreciation, \( ROI = rF \) and the positivity condition in an implicit form is \( rF(1-f) > w \) that is the same as for the annuity depreciation when \( F = 1 \) or \( g = r \). When \( r > g \) \((F > 0)\), the requirement for \( r \) is lower and when \( r < g \), it is higher. For the proportional depreciation, \( ROI = r(1+g)/(1+r) \) and \( NPM > 0 \) when \( r(1/(1+g)-w) > w \) that is for \( r \) higher than for the annuity depreciation when \( g < w/(1-f) \). It is higher than for the realization depreciation when \( g < F-1+w/(1-f) \). The annuity depreciation leads to \( NPM \) higher than the proportional depreciation when \( g < w/(1-f) \).

Table 3. Net profit margin (NPM) for different depreciation theories

A. Realization depreciation

\[
NPM = \frac{q\left[rF(1-f) - w\right]}{(1+r)(1+g) - qF} = \frac{q\left[rF(1-f) - w\right](1+g-q)(1+r)}{(1+r)(1+g) - qF(1+r-q)(1+g)}
\]

B. Proportional depreciation

\[
NPM = \frac{q\left[r((1-f)(1+g) - w) - w\right]}{(1+g)(1+r-q)}
\]

C. Annuity depreciation

\[
NPM = \frac{q\left[r(1-f) - w\right]}{(1+g)(1+r-q)}
\]
3.2.3 Net cash margin
The net cash margin $NCM$ can be calculated as follows:

$$NCM = \frac{CR(t) - CE(t) - I(t) - D(t) - T(t)}{CR(t)} - \frac{CM(t) - W(t)}{CR(t)}$$

where $CM(t) = CR(t) - CE(t)$ that is independent of the depreciation theory.

Table 4 shows the expressions of (20) derived for the depreciation theories. These expressions lead to complicated positivity conditions with respect to $r$. Equation (20) shows that $NCM$ depends on the depreciation theory only through $W(t)$ in the same way as $NQM$. Thus, if $r > 0$ and $g > -w/f$, the margin is the highest for the realization depreciation and the lowest for the proportional depreciation while the annuity depreciation gives values between these extremes. Similarly as above, the order of depreciation theory effects is reversed when $r < 0$. The lower the absolute value of $r$, the smaller is the difference between the ratios derived for different depreciations. If $r = 0$, the ratios are identical for the theories.

<table>
<thead>
<tr>
<th>A. Realization depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NCM = 1 - \frac{1 + g(1 - p)}{1 + g(1 - u)} \frac{1}{F} - \frac{q(w/F + fr)(1 + g)}{(1 + g(1 - u))(1 + r)(1 + g) - q}$</td>
</tr>
<tr>
<td>$= GCM - \frac{q(w/F + fr)(1 + g)}{(1 + g(1 - u))(1 + r)(1 + g) - q}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>B. Proportional depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NCM = 1 - \frac{1 + g(1 - p)}{1 + g(1 - u)} \frac{1}{F} - \frac{q(w(1 + r) + fr(1 + g))}{(1 + g(1 - u))(1 + r - q)}$</td>
</tr>
<tr>
<td>$= GCM - \frac{q(w(1 + r) + fr(1 + g))}{(1 + g(1 - u))(1 + r - q)}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>C. Annuity depreciation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$NCM = 1 - \frac{1 + g(1 - p)}{1 + g(1 - u)} \frac{1}{F} - \frac{q(w + fr)}{(1 + g(1 - u))(1 + r - q)}$</td>
</tr>
<tr>
<td>$= GCM - \frac{q(w + fr)}{(1 + g(1 - u))(1 + r - q)}$</td>
</tr>
</tbody>
</table>

3.2.4 Discussion
The inclusion of profit sharing items makes the analytical results complicated. This is because also net quick and cash margins are different for different theories. However, these net margins depend on the theory only through the profit-sharing items. Thus, it can be concluded that these financial flows are identical for different depreciations when $IRR = 0$. If $IRR > 0$, the realization depreciation leads to the highest margins while the proportional depreciation gives the lowest one. The annuity depreciation shows the values between these extremes. If $IRR < 0$ (as for failing firms), the order of depreciations is reversed. In that case, the accelerated realization depreciation is associated with the lowest value of flows. Therefore, expenditure and cash margins are not independent of depreciations if they are measured after profit-sharing items.

The target to cover profit-sharing items increases considerably the minimum required $IRR$. In this requirement, the rate of growth plays an important role. For the net expenditure margin, the positivity condition is associated with the highest $IRR$ when the proportional depreciation is used and when $g > 0$. The minimum requirement of $IRR$ is higher for the realization depreciation than for the annuity depreciation when $IRR > g$. When the golden rule (path) holds so that $IRR = g$, both depreciations show the same condition. Therefore, in empirical analyses it is important to pay
attention to the relationship between \( IRR \) and \( g \) when assessing comparability between firms using different depreciations.

The net profit margin is an important measure in financial statement analysis. When the firm applies the annuity depreciation, \( ROI = IRR \) without any bias. In this case, the net profit margin (retained earnings) is positive when \( IRR \) after taxes exceeds WACC. For the realization depreciation, the requirement for \( IRR \) depends on the relation between \( IRR \) and \( g \). If \( IRR > g \), the requirement is higher and vice versa. For the golden rule, the requirements are equal. For the depreciation theories, the relation between \( g \), WACC, and the rate of income tax is important for the positivity condition of net margins. For the net cash margin, this requirement is analytically complicated.

4. Empirical Illustration of the Theoretical Results

4.1 Sample and Estimation of the Parameters

In this section, the theoretical results will be illustrated by figures from failing (default) and non-failing (non-default) Finnish firms. The data are obtained from Suomen Asiakastieto Oy (http://www.asiakastieto.fi) for research purposes. The data include a sample of Finnish firms which have published annual financial statements in accounting years 2002-2003. The sample includes randomly selected 328 failing and 1358 non-failing firms. A firm was regarded as a failing (default) firm if it experienced a registered payment default after the end of 2003 but before 31st April 2005 (event period). In Finland, more than 40% of the payment defaults are private-judicial draft protests.

The model parameters are estimated intuitively because of the short-time series. In all, thirteen parameters are estimated from the last available accounting year and nine of them are used in numerical experiments. Total expenditure \( E(t) \) is calculated as the sum of current and fixed expenditure. Total revenue \( R(t) \) is measured by net sales. The rate of growth for total expenditure \( g^e \) and the rate of growth for total revenue \( g^r \) are calculated. The estimate of the steady growth \( g \) is calculated as the weighted average of these growth rates using total expenditure and total revenue as weights. The estimate of \( IRR \) or \( r \) is extracted from the relation between \( ROI \) and \( IRR \) assuming the neutral (proportional) depreciation so that \( r = ROI/(1+g-ROI) \). The lag parameter \( q \) is estimated from the relation between the parameter and the average lag \( K = q/(1-q) \). It is assumed that the average time lag for current expenses (purchases excluded) is 0.25 years, for purchases 0.5 years, and for depreciations 5 years. Then, a weighted average or \( K \) is calculated using expenses as weights and \( q = K/(1+K) \).

The rate of accounts payable \( p \) is calculated as the ratio of accounts payable to expenditure. The rate of accounts receivable \( u \) is estimated as the ratio of accounts receivable to net sales. The debt to assets ratio \( DAR \) or \( b \) is approximated by the ratio of total debt (accounts payable excluded) to total assets. The effective income tax rate \( f \) is got by dividing income taxes by \( EBIT \) minus interest payments. The interest rate \( i \) is calculated as the ratio of interest payments divided by the total debt (accounts payable excluded) on the beginning of the year balance. The rate of dividends \( d \) is estimated by the ratio of approximated dividends by the equity on the beginning of the year basis. The statistical estimations are all carried out by the SPSS.

4.2 Descriptive Statistics

Table 5 shows descriptive statistics for the parameters of the model. The distributions for the estimates are generally very skew. Therefore, the median is used to refer to a typical firm. The median of \( E(t) \) shows that that a typical firm in both groups is very small. The difference in \( E(t) \) between failing and non-failing firms is not statistically significant whereas that in \( R(t) \) is, referring to the lower level of revenue finance in failing firms. The median growth rates \( g^e \) and \( g^r \) are about 3% for the non-failing firms while they are negative for the failing firms. In addition, \( g^r \) is less than \( g^e \) which implies that the steady-state assumption does not hold exactly for either group. However, on average the growth process is close to steady since the relation of growth components is close.
the unity. The median IRR for the failing firms is only about 4% that does not cover the cost of capital. For the median non-failing firm, it is as high as 17%.

The median rate of accounts payable \( p \) for the failing firms exceeds 18% but is only 8% for the non-failing group. Thus, failing firms have used this finance significantly more. The median rate of accounts receivable \( u \) is about 6% for both groups. The median DAR \( (b) \) for the failing firms is about 90% while it is only 52% for the non-failing ones. This shows that failing firms are deeply indebted and financially distressed before the event (payment default). The median tax rate \( f \) is 29% for the non-failing firms corresponding approximately to the company tax rate in legislation. Because of losses, this rate is zero for a median failing firm. For the failing firms, the median rate of interest \( i \) is over 5% but for the non-failing ones only about 2%. The median rate of dividend \( d \) is 8% for the non-failing firms but zero for the failing ones showing that a median failing firm does not pay dividends (about one year) before default.

Table 5. Descriptive statistics of the financial flow model parameters

<table>
<thead>
<tr>
<th>Parameter $</th>
<th>Failing firms (N = 328)</th>
<th>Non-failing firms (N= 1358)</th>
<th>Chi-Square #</th>
</tr>
</thead>
<tbody>
<tr>
<td>( E(t) ) (Euro)</td>
<td>Mean 1385900.0 Std. Dev. 10093100.0 Median 305916.0</td>
<td>Mean 3942300.0 Std. Dev. 35075700.0 Median 354388.5</td>
<td>0.2420</td>
</tr>
<tr>
<td>( R(t) ) (Euro)</td>
<td>Mean 1366640.5 Std. Dev. 10640000.0 Median 276813.0</td>
<td>Mean 4008823.7 Std. Dev. 35630000.0 Median 379321.5</td>
<td>0.0080</td>
</tr>
<tr>
<td>( g' )</td>
<td>0.0881 0.5218 -0.0284</td>
<td>0.0931 0.4115 0.0299</td>
<td>0.0040</td>
</tr>
<tr>
<td>( g )</td>
<td>0.0844 0.5070 -0.0448</td>
<td>0.0839 0.3792 0.0273</td>
<td>0.0000</td>
</tr>
<tr>
<td>( (1+g')(1+g) )</td>
<td>1.0582 0.4215 0.9930</td>
<td>1.0542 0.4047 0.9956</td>
<td>0.8540</td>
</tr>
<tr>
<td>( r )</td>
<td>0.1011 0.4046 0.0433</td>
<td>0.2922 0.4650 0.1744</td>
<td>0.0000</td>
</tr>
<tr>
<td>( q )</td>
<td>0.3391 0.0798 0.3261</td>
<td>0.3446 0.0863 0.3269</td>
<td>0.8540</td>
</tr>
<tr>
<td>( p )</td>
<td>0.3082 0.3216 0.1864</td>
<td>0.1684 0.2498 0.0816</td>
<td>0.0000</td>
</tr>
<tr>
<td>( u )</td>
<td>0.0966 0.1401 0.0632</td>
<td>0.0790 0.1031 0.0554</td>
<td>0.1570</td>
</tr>
<tr>
<td>( b )</td>
<td>0.8134 0.2213 0.8980</td>
<td>0.5113 0.2906 0.5150</td>
<td>0.0000</td>
</tr>
<tr>
<td>( f )</td>
<td>0.1216 0.2163 0.0000</td>
<td>0.2018 0.2043 0.2878</td>
<td>0.0000</td>
</tr>
<tr>
<td>( i )</td>
<td>0.0759 0.0851 0.0532</td>
<td>0.0407 0.0766 0.0220</td>
<td>0.0000</td>
</tr>
<tr>
<td>( d )</td>
<td>0.0618 0.1990 0.0000</td>
<td>0.1431 0.2353 0.0754</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Legend:
\$ = parameter definitions:
\( E(t) \) = expenditure
\( R(t) \) = revenue
\( g' \) = growth rate of expenditure
\( g \) = growth rate of revenue
\( g \) = weighted rate of growth
\( (1+g')(1+g) \) = relation between rates of growth components (steady state measure)
\( r \) = internal rate of return
\( q \) = lag parameter
\( p \) = rate of accounts payable
\( u \) = rate of accounts receivable
\( b \) = debt to total assets
\( f \) = (effective) tax rate
\( i \) = interest rate
\( d \) = dividend rate
\# = significance level of the Chi-Square statistic on the equity of medians
4.3 Groups of Failing Firms

Table 5 only presents the mean and median of the model variables for failing firms. These firms can be described more efficiently when clustering the firms into independent groups. Therefore, the factor analysis was applied to the fourteen parameter values of failing firms and the orthogonal Varimax rotation was used to extract factors ($N=328$). For the final solution, six factors were selected on the basis of the eigenvalues. The first factor accounts for 21% of the total variation but the sixth factor accounts only for 8%. In all, the six-factor solution accounts for 72% of the total variation giving a good summary of the parameter values.

The largest standardized factor score was used to classify each failing firm into one of the six groups. Thus, the typology is based on the characteristic of the firm that is most outstanding. Table 6 presents descriptive statistics for the six groups. The failing firms are distributed quite equally between the groups except for group 2 including only 10% of the sample. First, the firms in group 1 are characterized by exceptionally high growth rates ($g'$, $g''$, and $g$). The median firm has a negative IRR ($r$) and it does not pay taxes or dividends before default. Obviously, the cause for default is based on unprofitable rapid growth. Thus, this type can be called “unprofitable rapid growth firm”. This kind of unprofitable growth process inevitably leads to insufficiency of revenue finance. This type of firm is indebted but not as heavily as the median firms in most of groups.

Table 6. Descriptive statistics for different groups of failing firms

<table>
<thead>
<tr>
<th>Variable$^8$</th>
<th>Group 1</th>
<th>Group 2</th>
<th>Group 3</th>
<th>Group 4</th>
<th>Group 5</th>
<th>Group 6</th>
<th>Chi-Square$^9$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nr. of firms</td>
<td>61</td>
<td>33</td>
<td>65</td>
<td>62</td>
<td>60</td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>% of firms</td>
<td>0.1860</td>
<td>0.1006</td>
<td>0.1982</td>
<td>0.1890</td>
<td>0.1829</td>
<td>0.1433</td>
<td></td>
</tr>
<tr>
<td>$E(t)$ (Euro)</td>
<td>368284</td>
<td>645770</td>
<td>359698</td>
<td>201751</td>
<td>191486</td>
<td>346677</td>
<td>0.0390</td>
</tr>
<tr>
<td>$R(t)$ (Euro)</td>
<td>319085</td>
<td>587000</td>
<td>372444</td>
<td>206757</td>
<td>185634</td>
<td>217238</td>
<td>0.0060</td>
</tr>
<tr>
<td>$g'$</td>
<td>0.7503</td>
<td>-0.1070</td>
<td>-0.0303</td>
<td>-0.0488</td>
<td>-0.2901</td>
<td>-0.1099</td>
<td>0.0000</td>
</tr>
<tr>
<td>$g''$</td>
<td>0.7943</td>
<td>-0.1322</td>
<td>-0.0493</td>
<td>-0.0625</td>
<td>-0.1039</td>
<td>-0.1464</td>
<td>0.0000</td>
</tr>
<tr>
<td>$g$</td>
<td>0.7832</td>
<td>-0.1197</td>
<td>-0.0213</td>
<td>-0.0634</td>
<td>-0.1807</td>
<td>-0.1051</td>
<td>0.0000</td>
</tr>
<tr>
<td>$(1+g')(1+g')$</td>
<td>0.9665</td>
<td>0.9639</td>
<td>0.9778</td>
<td>0.9871</td>
<td>1.2501</td>
<td>0.8958</td>
<td>0.0000</td>
</tr>
<tr>
<td>$r$</td>
<td>-0.0467</td>
<td>-0.0403</td>
<td>0.2453</td>
<td>0.0764</td>
<td>-0.0458</td>
<td>0.0152</td>
<td>0.0000</td>
</tr>
<tr>
<td>$q$</td>
<td>0.3187</td>
<td>0.3160</td>
<td>0.3231</td>
<td>0.3121</td>
<td>0.4184</td>
<td>0.2899</td>
<td>0.0000</td>
</tr>
<tr>
<td>$p$</td>
<td>0.1078</td>
<td>0.1333</td>
<td>0.0870</td>
<td>0.5217</td>
<td>0.1714</td>
<td>0.3101</td>
<td>0.0000</td>
</tr>
<tr>
<td>$u$</td>
<td>0.0515</td>
<td>0.0543</td>
<td>0.0517</td>
<td>0.0448</td>
<td>0.0520</td>
<td>0.1839</td>
<td>0.0000</td>
</tr>
<tr>
<td>$b$</td>
<td>0.8780</td>
<td>0.9320</td>
<td>0.6970</td>
<td>0.9580</td>
<td>0.9300</td>
<td>0.9540</td>
<td>0.0000</td>
</tr>
<tr>
<td>$f$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.2973</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
<tr>
<td>$i$</td>
<td>0.0383</td>
<td>0.0517</td>
<td>0.0472</td>
<td>0.1375</td>
<td>0.0530</td>
<td>0.0364</td>
<td>0.0000</td>
</tr>
<tr>
<td>$d$</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0875</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

Legend:
$^8$ = Refer to Table 5

Second, the median firm in group 2 is larger than the median size in other groups. It is heavily indebted and has negative $r$ and $g$. It does not pay any taxes or dividends before default. This type of default firm can thus be called “unprofitable larger firm”. Third, group 3 is the largest group including 22% of the sample firms. The median firm has many good financial characteristics. It is quite profitable, not heavily indebted, has a low rate of accounts payable, and it pays taxes and dividends. The only negative characteristic is negative growth in terms of $g'$, $g''$, and $g$. The steady-state factor $(1+g')(1+g')$ is clearly below unity implying that profitability is deteriorating. Thus, this type of default firm can be entitled as “profitable negative growth firm”.

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Fourth, the median firm in group 4 has positive profitability $r$ but it is heavily indebted in terms of $b$ and especially $p$. Consequently, the cost of debt $i$ is very high and the type can be called “high cost-of-debt firm”. Fifth, the median firm of group 5 has very low negative $g'$ and thus, in spite of negative $g$, a very high steady-state factor. This kind of firm has a serious break in its growth process (especially in that of expenditure) and it can be entitled as “non-steady expenditure firm”. It has also an exceptionally high lag parameter $q$ which refers to remarkable long-term investments. Sixth, the median firm in group 6 has $r$ close to zero but it is heavily indebted both in terms of $b$ and $p$. It has also an exceptionally high rate of accounts receivable $u$. It has negative growth rates, especially in terms of $g'$ which has led to a very low steady-state factor and obviously to high $u$. Therefore, this type can be called “non-steady revenue firm”.

4.4 Flow Ratios in Failing Firms

4.4.1 Six different types

The second column of Table 7 presents the financial flow variables for the median non-failing firm. Because $r$ is high and $g$ very low, it shows a good level of revenue finance and all gross and net margins are positive. The margins do not remarkably differ from each other with respect to the depreciation theory due to low $g$ (in spite of high $r$). However, low $g$ weakens the reliability of $ROI$ leading to a lower value for the proportional depreciation. Because of low $g$, quick and cash margins do not remarkably differ from each other while profit margins are higher due to high $r$.

The third column shows the margins for the median failing firm. Because of low $r$, the net profit margins are negative. However, negative $g$ makes all quick and cash margins positive although being close to zero. Its revenue finance is low but sufficient to finance interest expenses (taxes and dividends are zero). The depreciation theory has not a remarkable effect on the ratios due to low $g$ and $r$.

The fourth column reports the margins for “unprofitable rapid growth firm” (group 1). Because of negative $r$ and very high $g$, all the ratios are negative. Especially, all quick and cash margins are extremely low due to high $g$. The extensive difference between $r$ and $g$ remarkably affects the reliability of $ROI$ since the effect of $g$ is different for different depreciation theories. The fifth column includes figures for “unprofitable larger firm” (group 2). It has negative $r$ but even lower $g$. This has led to that profitability margins are negative whereas quick and cash margins are positive. Net quick and cash margins are very close to zero. The effect of depreciation theory on profitability margins is insignificant due to low $r$.

The sixth column deals with figures for “profitable negative growth firm” (group 3) which shares many characteristics of non-failing firms but has negative $g$. This type has high $r$ which together with negative $g$ makes all margins very high. For this type, revenue finance is sufficient to pay average taxes and dividends. This type does not show any other signs of failure than low $g$ which makes the effect of depreciation on margins negligible. However, the reliability of $ROI$ is low due to the large difference between $r$ and $g$. The fourth type “high cost-of-debt firm” (group 4) has positive $r$ less than 10\% but negative $g$. It is very indebted and has remarkable interest cost. Therefore, net profit margins are negative. Its net quick ratios are about zero but net cash margins are negative which is due to high $p$ and negative $g$. The effect of depreciation on margins is small due to low $g$.

The fifth type of failing firm is “non-steady expenditure firm” (group 5). It has negative $r$ and very low negative $g$. Because of extremely low $g$, all quick and cash margins are very high (higher than for average non-failing firms). However, all profit margins are negative due to negative $r$. The effect of depreciation is not strong because of low $r$. The sixth type is called “non-steady revenue firm” (group 6) having $r$ close to zero but negative $g$. Consequently, its profit margins do not remarkably differ from zero (gross margins are positive and net margins negative) but net quick and cash margins are positive. Because of low $r$, the effect of depreciation is small.
### Table 7. Financial flow ratios in six groups of failing firms

<table>
<thead>
<tr>
<th>Non-failing firms</th>
<th>Failing firms</th>
<th>Ratios based on the median values of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Median</td>
<td>Median</td>
</tr>
<tr>
<td><strong>1. Gross margins</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gross quick margin</td>
<td>0.0560</td>
<td>0.0335</td>
</tr>
<tr>
<td>Gross profit margin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Realization depreciation</td>
<td>0.0644</td>
<td>0.0205</td>
</tr>
<tr>
<td>Proportional depreciation</td>
<td>0.0673</td>
<td>0.0197</td>
</tr>
<tr>
<td>Annuity depreciation</td>
<td>0.0652</td>
<td>0.0202</td>
</tr>
<tr>
<td>Gross cash margin</td>
<td>0.0538</td>
<td>0.0292</td>
</tr>
<tr>
<td><strong>2. Net margins</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Net quick margin</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Realization depreciation</td>
<td>0.0155</td>
<td>0.0099</td>
</tr>
<tr>
<td>Proportional depreciation</td>
<td>0.0141</td>
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<td>Annuity depreciation</td>
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<td>0.0101</td>
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<tr>
<td>Net profit margin</td>
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<td></td>
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<td>Realization depreciation</td>
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<td>-0.0014</td>
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<td>-0.0036</td>
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<td>Annuity depreciation</td>
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<td>-0.0021</td>
</tr>
<tr>
<td>Net cash margin</td>
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<td></td>
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<td>Realization depreciation</td>
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<td>0.0060</td>
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<td>0.0070</td>
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<td><strong>3. Return on investment ratio</strong></td>
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<td>Realization depreciation</td>
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<td>Annuity depreciation</td>
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### 4.4.2 Discussion

Empirical evidence shows that non-failing firms typically show high IRR and low $g$. Thus, they have sufficient revenue finance and earnings to pay taxes and dividends, and still have some percent of revenues as retained earnings. For a typical non-failing firm, all net margins are positive. The effect of depreciation theory on margins is small because of low $g$. However, this effect is remarkable on ROI because of the large difference between IRR and $g$. Thus, there may be reliability problems when measuring profitability but not when analyzing flow ratios. In a typical failing firm, IRR may be positive but small and $g$ is usually negative. This kind of average failing firm may due to negative $g$ have revenue finance enough to pay interest payments in spite of high DAR. However, it usually does not pay taxes or dividends because of low IRR. For a typical failing firm, net margins are close to zero or negative. The effect of depreciation theory on margins and
ROI is small because of low IRR and low g. Thus, reliability problems may not be serious in average failing firms.

Evidence shows that there may be at least six different types of failing firms. In five of the six types, the median firm has negative g. However, one type of firms shows very high g and negative IRR leading to serious difficulties with revenue finance. For this type, all net margins are negative and very low. The effect of depreciation on margins is small due to low IRR but the difference between g and IRR makes the effect on ROI remarkable. This type is not rare representing almost 20% of failing firms. In four of the six types, IRR is typically negative or very close to zero. However, one type representing 20% of failing firms shows IRR higher than for an average non-failing firm. It also pays taxes and dividends. It is a look-a-like non-failing firm but shows negative g and high DAR. For this type, all net margins are positive and higher than for a typical non-failing firm. Because of low g, the effect of depreciation on margins is small but remarkable on ROI. In five of the six types, the median firm shows g < IRR which makes F > 1. Negative g is for most failing firms a way to increase revenue finance that otherwise would be very low due to low IRR. For most types of failing firms, g and IRR are low making the effect of depreciation on margins and ROI small.

5. Concluding Remarks

5.1 Model Concepts

Financial ratio analysis suffers from the lack of theoretical research especially for analyzing failing firms behaving abnormally. The purpose of this study was to develop a theoretical framework based on assumptions of steady growth, constant IRR, and constant lag between flows of expenditure and revenue. It shows that revenue finance is higher, the higher is IRR and the lower is g. If IRR = g, periodic revenue equals expenditure. If IRR is low, the rate of finance is low. In addition, if g is high, the firm may be faced by a serious financial distress. The industry effect is reflected by the time lag. For a profitable but slow-growing firm, revenue finance is increasing in lag. However, for a fast-growing firm with low IRR, the effect is reversed. If IRR = g, revenue finance is independent of the industry effect. If IRR is close to g, the industry effect is negligible.

Different depreciation theories lead to different expense and asset concepts when IRR or g differs from zero. The lower the absolute value of IRR or g, the weaker is the effect of depreciation. Therefore, for a failing firm showing low IRR or g, expense and asset concepts may not be sensitive to the depreciation method. However, when more profitable firms are analyzed, this effect may be significant. The effect of depreciation is conditional to the positivity of IRR. When IRR is positive, an accelerated depreciation gives the highest depreciation and the lowest assets. In this situation, a neutral method gives lowest depreciation and highest assets. When IRR is negative, the results are opposite. Thus, for failing firms with negative IRR, an accelerated method may lead to the lowest depreciation. Furthermore, if g is very small, all theories give depreciations close to expenditure. Thus, in a slow-growing failing firm, the effect of depreciation is expected to be negligible.

5.2 Financial Margins

The results show that different gross margins are sensitive to different situations. First, quick margin is directly related to the rate of revenue finance reflecting the difference between IRR and g. It is zero if the firm follows the golden rule. The relation between IRR and g determines the sign of the margin. For fast-growing firms with lower profitability it is negative and positive if the opposite holds. The industry effect depends on the sign of the margin. If the margin is negative for a failing firm, longer lag tends to lead to lower margins and vice versa. Second, profit margin is sensitive to IRR supporting its validity in profitability measurement. It is positive for positive IRR. Hence, for a failing firm with a negative IRR, the profit margin is negative for all depreciation theories. The
validity of the ratio is not strongly distorted by growth. However, the margin is largely determined by the industry effect. Thus, longer lag tends to lead to higher margins. The differences for depreciation theories largely depend on \( g \). For a slow-growing firm, the effect of depreciation theory is small. However, in fast-growing firms it is more relevant deteriorating reliability.

Finally, cash flow is strongly affected by the growth of the firm. If \( g = 0 \), cash margin equals quick margin and is positive for positive \( IRR \). The positivity of gross cash margin largely depends on the proportions of accounts payable and accounts receivable in addition to \( IRR \) and \( g \). If the proportions are equal, cash margin is positive when \( IRR > g \). If \( IRR > g \), longer lag tends to lead to higher cash flows, and vice versa. Typically, cash flows suffer from fast growth. For a failing firm with a low \( IRR \), the cash flow margin is usually very low.

Net quick and cash margins depend on the depreciation theory only through the profit sharing items. Thus, these financial flows are identical for different depreciations when \( IRR = 0 \). If \( IRR > 0 \), the realization depreciation leads to highest margins while the proportional depreciation gives lowest ones. The annuity depreciation shows the values between these extremes. If \( IRR \) is negative (as for failing firms), the order of depreciations is reversed. For all depreciations, net profit margin is very sensitive to the industry effect in the same way as the gross profit margin. If the annuity depreciation is applied, \( IRR = ROI \) emphasizing validity of \( ROI \) in profitability measurement. However, for other depreciations, this equality does not hold.

5.3 Empirical Figures

Empirical evidence shows that non-failing firms typically show high \( IRR \) and lower \( g \). Thus, revenue finance is sufficient to make net margins positive. The effect of depreciation theory on margins is small because of low \( g \). However, the effect on the reliability of \( ROI \) may be significant due to the large difference between \( IRR \) and \( g \). In average failing firms, \( IRR \) may be positive but low and \( g \) is usually negative. For this kind of failing firm net margins are close to zero or negative. The effect of depreciation on margins and \( ROI \) is small because of low \( IRR \) and low \( g \). Therefore, reliability problems in profitability measurement may not be very relevant in average failing firms.

Evidence shows that there are several types of failing firms. In most types, the median firm has negative \( g \). However, one type grows very fast showing negative \( IRR \). For this type, all net margins are negative and very low. The effect of depreciation is small due to low \( IRR \) but the reliability of \( ROI \) is deteriorated by the difference between \( g \) and \( IRR \). In most types, \( IRR \) is negative or very close to zero. However, one type reports very high \( IRR \) but shows negative \( g \). For this type, all net margins are positive and higher than for a typical non-failing firm. Because of low \( g \), the effect of depreciation on margins is small but at the same time it is remarkable on \( ROI \). In most types, the median firm shows \( g < IRR \) which makes the rate of revenue finance less than unity. For most types of failing firms, \( g \) and \( IRR \) are low making the effect of depreciation on margins and \( ROI \) small.

5.4 Implications

In summary, the main findings of the study can be summarized in the following implications for financial statement analysis:

1. Depreciations have a relevant effect of financial flow margins only when \( g \) is high and \( IRR \) is not low. Depreciations remarkably weaken the reliability of \( ROI \) only when the difference between \( IRR \) and \( g \) is remarkable.

2. Effect of industry (reflected by time lag) on flow margins is very strong but does not affect the positivity condition of \( IRR \) for margins. There is weak comparability in the absolute level of margins between firms with different time lag. However, positivity of the margins can serve as a comparable norm across different firms.
(3) Failing firms often show negative growth which makes net quick and cash margins positive and comparable with margins of non-failing firms. Negative growth in failing firms is usually associated with low profitability. Low profitability and negative growth make the effect of depreciation on margins small.

(4) Failing firms with negative growth may sometimes show high profitability which makes them difficult to identify as a failing firm. Large difference between growth and profitability weakens the reliability of \( ROI \).

(5) Failing firms with low profitability and very high growth are characterized by negative net quick, profit, and cash margins. Large difference between growth and profitability weakens the reliability of \( ROI \).

(6) Non-failing firms typically show good profitability and low positive growth so that all net margins tend to be positive. Low growth rate makes the effect of depreciation on margins small. The difference between growth and profitability does not seriously weaken the reliability of \( ROI \).

References