Some Properties of Codes with Infinite Deciphering Delay

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Abstract: In 2013, Tommi Lehtinen and Alexander Okhotin proved that if $X$ is a code, then it has infinite deciphering delay if and only if there exist $x, y, z \in A^+$ with $x, xy, yz, zy \in X^+$ and $y, z \notin X^*$. In this paper, we give a sufficient and necessary condition for codes with infinite deciphering delay. Then, we construct two kinds of three-element codes with infinite deciphering delay.

Keywords: Infinite Deciphering Delay, Prefix Graph, Construction, Injective Morphism

1 Introduction

A very important property required for codes is that decoding a transmitted message is possible before its complete reception. This quality is satisfied in particular by codes with a finite deciphering delay. Codes with finite deciphering delay have important application in the field of information theory and coding theory (see [1,4-5]). In 2001, Jean Néraud and Carla Selmi proved that if $X$ is a non-complete code with a finite deciphering delay, then there exists an uncompletable word $w$ of length $O(m^2d^2)$, where $d$ stands for the delay and $m$ stands for the length of the longest words in $X$ (see [6]). In 2014, Lila Kari and Stavro Knostantindis considered the problem of deciding maximality of a regular language with respect to deciphering delay 1 and
transducer-based property, such as suffix code, overlap-free language and error-detection properties (see [7]).

However, researches on codes with infinite deciphering delay help us to know them from another aspect, and they have important application in information theory. For example, the characterization of codes with infinite deciphering delay is used to construct examples of languages witnessing the non-closure under certain morphism (see [2]). In this paper, we give some properties of codes with infinite deciphering delay. First, we show that if $X$ is a code, then it has infinite deciphering delay if and only if there exist three paths in the prefix graph of $X$. Second, we prove that codes with infinite deciphering delay preserve injective morphism. Finally, we give two sufficient conditions on three-element codes with infinite deciphering delay. The proofs lead explicit constructions of these three-element codes.

2 Preliminaries

Let $A$ be a finite set, which is called an alphabet. An element $a \in A$ is called a letter. A word $w$ on the alphabet $A$ is a finite sequence of letters in $A$, that is $w = a_1 a_2 K a_n$ where $a_i \in A$ and $i = 1, K , n$. The set of all words on the alphabet $A$ is denoted by $A^*$ and is equipped with the associative operation defined by $(a_i a_2 K a_n) \cdot (b_i b_2 K b_m) = a_i a_2 K a_n b_i b_2 K b_m$. The empty sequence is called the empty word and is denote by $\varepsilon$. It is the neutral element for concatenation. Then $A^*$ is a free monoid generated by $A$. Let $A^+ = A^* \setminus \{ \varepsilon \}$. The length $|w|$ of a word $w = a_1 a_2 K a_n$ with $a_i \in A, i = 1, K , n$ is the number $n$ of letters in $w$. So $|\varepsilon| = 0$. A nonempty word $w$ is a primitive word, if $w$ is not a power of any other nonempty word. Let $Q$ be the set of all primitive words over $A$. Let $A, B$ be alphabets, and $Z \subseteq A^+, Y \subseteq B^+$ be two codes. Then the codes $Y$ and $Z$ are composable if there is a bijection from $B$ onto $Z$. If $\beta$ is such a bijection, then $Y$ and $Z$ are called composable through $\beta$. Then $\beta$ defines a morphism from $B^+$ into $A^+$ which is injective since $Z$ is a code. The set $X = \beta(Y) \subseteq Z^+ \subseteq A^*$ is obtained by composition of $Y$ and $Z$ (by means of $\beta$). We denote by $X = Y \circ_B Z$ or $X = Y \circ_C Z$, when the context permits it.

Let $X$ be a finite set of words over alphabet $A$. We define a graph $G_X$ of $X$, which is called the prefix graph of $X$ as follows. The vertices of $G_X$ are the nonempty prefixes of words in $X$, and there is an edge from $s$ to $t$ if and only if one of the two following situation occurs: either $st \in X$ or $sx = t$ for some $x \in X$. Edges of the first type are called crossing, and edges of the second type are called extending. A crossing edge $(s, t)$ is labeled with the word $t$, and an extending edge $(s, t)$ with $sx = t$ is labeled with $x$. Two factorizations $(x_1, x_2, K, x_n)$ and $(y_1, y_2, K, y_m)$ of a word are disjoint if $x_1 x_2 K x_n = y_1 y_2 K y_m$ and $x_1 x_2 K x_i \neq y_1 y_2 K y_j$ for any $1 \leq i < n, 1 \leq j < m$. A nonempty subset $X$ of $A^*$ is said to have finite verbal deciphering delay if there exists an integer $d \geq 0$ such that for any $x, x', y' \in X$ and $y \in X^d$, $xy \leq_p x'y'$ implies $x = x'$. If no such integer exists, the set $X$ has infinite deciphering delay.

Lemma 2.1 [21] Let $X \subseteq A^*$ be a code. Then it has infinite deciphering delay if and only if there
exist \( x, y, z \in A^+ \) with \( x, xy, yz, zy \in X^* \) and \( y, z \notin X^* \).

**Lemma 2.2** \(^1\) There is a path of length \( n (\geq 1) \) from \( s \) to \( t \) in the prefix graph of \( X \) if and only if there exist \( x_1, K, x, y_1, K, y, t \) in \( X \) such that \( s y_1 K y_1 t = x_1 K x \) or \( s y_1 K y_1 t = x_1 K x \) are disjoint factorizations with \( k + l = n \), where \( s \) is a prefix of \( x_1 \) or \( s \) is a prefix of \( t \) if \( k = 0 \). The label of the path is \( y_1 K y_1 t \) in the first case and \( y_1 K y_1 t \) in the second case. The first (second) case occurs if and only if the path contains an odd (even) number of crossing edges.

**Lemma 2.3** \(^1\) The nonempty set \( X \subseteq A^+ \) is a code if and only if none of the set \( U_n \) contains the empty word, where \( U_1 = ( X^{-1} X ) - \{ \varepsilon \} \) and \( U_{n+1} = X^{-1} U_n U^{-1} X \) when \( n \geq 1 \).

**Lemma 2.4** \(^1\) Let \( \alpha : A^* \to C^* \) be an injective morphism. If \( X \) is a code over alphabet \( A \), then \( \alpha(X) \) is a code over \( C \). If \( Y \) is a code over \( C \), then \( \alpha^{-1}(Y) \) is a code over \( A \).

**Lemma 2.5** \(^3\) If \( u v = v z \) where \( u, v, z \in A^+ \) and \( u \neq \varepsilon \), then \( u = x y, v = (x y)^k x, z = y x \) for some \( x, y \in A^+ \) and \( k \geq 0 \).

**Lemma 2.6** \(^3\) If \( u v = v u \) where \( u \neq \varepsilon, v \neq \varepsilon \), then \( u \) and \( v \) are powers of a common word.

### 3 Infinite deciphering delay

In this section, we give a sufficient and necessary condition for codes with infinite deciphering delay, then construction methods are given for three-element codes with infinite deciphering delay. We first prove a lemma which will be used in the later results.

**Lemma 3.1** Let \( X \subseteq A^+ \) be a code. If \( s y_1 K y_1 t = x_1 K x_m \), where \( n \geq 0, m \geq 1 \), \( x_1, K, x, y_1, K, y, t \in A^+ \), \( s <_p x_1 \), \( t \in A^* \), and \( y_1 K y_1 t \notin X^* \). Then two factorizations \((s, y_1 K, y_1 t)\) and \((x_1 K, x_m)\) are disjoint.

Proof: We prove the result by induction on \( n + m \). If \( n + m = 1 \), then \( st = x_1 \). Since \( y_1 K y_1 t \notin X^* \), then \( t \neq \varepsilon \). Thus \((s, t)\) and \((x_1)\) are disjoint. Assume the result holds for \( n + m \leq k - 1 \). If \( n + m = k \), let \( s y_1 K y_1 t = x_1 K x_m \). Since \( y_1 K y_1 t \notin X^* \) then \( |t| \neq |x_m| \). Then one of the following two cases holds.

1. If \( |t| > |x_m| \), there exists \( t_i \in A^+ \) such that \( t_i x_m = t \). So \( s y_i K y_i t_i = x_i K x_{m-1} \), and \( y_1 K y_1 t \notin X^* \). Now \( m + n - 1 \leq k - 1 \). By the induction assumption, we know two factorizations \((s, y_1 K, y_1 t)\) and \((x_1 K, x_{m-1})\) are disjoint. Thus two factorizations \((s, y_i K, y_i t)\) and \((x_i K, x_{m-1})\) are disjoint because \( x_m \) is a proper suffix of \( t \).

2. If \( |t| < |x_m| \), then \( s y_i K y_i t = x_i K x_{m-1} \), \( y_i = w v \) and \( w v x_i K y_i t = x_m \) for some \( w, v \in A^* \) and \( 1 \leq i \leq n \). In fact \( w, v \neq \varepsilon \). Suppose \( w = \varepsilon \). Then \( y_i y_{i+1} K y_{i+1} t = x_m \in X \).
Hence \( y_1K y_{i+1}K y_m \neq y_1K y_i x_m \in X^* \), which is a contraction. By the same way, we know \( v \neq \epsilon \). Suppose that \( y_1K y_{i+1}w \in X^* \), then \( y_1K y_{i+1} w v y_{i+1}K y_m = y_1K y_{i+1} w x_m \), which is a contraction. Hence \( y_1K y_{i+1} w \notin X^* \). Since \( sy_{i+1}K y_{i+1} = x_iK x_{m-1} \), by the induction assumption, two factorizations \((s, y_1K, y_{i+1}, w)\) and \((x_iK, x_{m-1})\) are disjoint. Thus two factorizations \((s, y_1K, y_n,t)\) and \((x_iK, x_m)\) are disjoint because \( vy_{i+1}K y_m = x_m \). \( \square \)

**Theorem 3.2** Let \( X \subseteq A^+ \) be a code. Then it has infinite deciphering delay if and only if there exist three paths \((z, y'), \ (y', z)\) and \((x, y')\) in the prefix graph of \( X \), where \( x, y', z \in A^* \) and \( X \notin X^* \). The labels of these three paths are \( y, z \) and \( y \), respectively, and \( y, z \notin X^* \). Each path contains an odd number of crossing edges.

**Proof:** \(( \Rightarrow \) Let \( X \) is a code with infinite deciphering delay. By lemma 2.1, then \( x, y, z \in A^* \) and \( y, z \notin X^* \).

Since \( yz \in X^* \) and \( y, z \notin X^* \), then \( y = u_1K u_n y' \), \( z = z'v_1K v_m \) and \( y'z' \in X \) for some \( u_1K , u_n, v_1K, v_m \in X \), \( y', z' \in A^* \). Hence \( y' < y \) and \( z' < z \). Since \( zy \in X^* \), then \( y = \tilde{y} \tilde{u}_1K \tilde{u}_p \), \( z = v_1K v_q z \) and \( z'y \in X \) for some \( u_1K , u_p, \tilde{u}_1K, v_q \in X \), \( y, z \in A^* \). So \( \tilde{z} < y \), \( z < y \). Since \( xy \in X^* \), then \( y = y'v_1K v', z = u_1K u_p x \) and \( x'y \in X \) for some \( u_1K , u_p, v_1K, v' \in X \), \( x', x \in A^* \). Hence \( y' < z \), \( z < x \) and \( X \notin X^* \).

(1) Since \( y = u_1K u_n y' = \tilde{y} \tilde{u}_1K \tilde{u}_p \), then \( z = u_1K u_n y' = z'yv_1K v_m \). Let \( s_1 = \tilde{z} \), \( t_1 = y', u_i = y^{(i)}', i = 1, K, n \), \( x^{(1)}_1 = z'y \) and \( x^{(1)}_{j+1} = \tilde{u}_j, j = 1, K, p \). Hence we have \( s_1y_1K y^{(1)}_1t_1 = x^{(1)}_1K x^{(1)}_{m_1} \). By lemma 3.1, two factorizations \((s_1, y^{(1)}_1K, t_1)\) and \((x^{(1)}_1K, x^{(1)}_{m_1})\) are disjoint, where \( n_1 = n, m_1 = p + 1 \).

(2) Since \( z = v_1K v_q z = z'v_1K v_m \), then \( y'v_1K v_q z = y'z'v_1K v_m \). Let \( s_2 = y' \), \( t_2 = z, v_j = y^{(2)}_i, i = 1, K, q \), \( x^{(2)}_1 = y'z' \) and \( x^{(2)}_{j+1} = v_j, j = 1, K, m \). So \( s_2y_2K y^{(2)}_2t_2 = x^{(2)}_1K x^{(2)}_{m_2} \). By lemma 3.1, two factorizations \((s_1, y^{(1)}_1K, t_1)\) and \((x^{(1)}_1K, x^{(1)}_{m_1})\) are disjoint, where \( n_2 = q, m_2 = m + 1 \).

(3) Since \( y = u_1K u_n y' = y'v_1K v' \), then \( x = \tilde{x} u_1K u_n y' = \tilde{x}y'v_1K v' \). Let \( s_3 = \tilde{x} \), \( t_3 = y', u_i = y^{(3)}_i, i = 1, K, n \), \( x^{(3)}_1 = \tilde{x}y' \) and \( x^{(3)}_{j+1} = v_j, j = 1, K, r \). Thus we have \( s_3y_3K y^{(3)}_3t_3 = x^{(3)}_1K x^{(3)}_{m_3} \). By lemma 3.1, two factorizations \((s_3, y^{(3)}_1K, t_3)\) and
Let \( x_i \) be an injective morphism. If \( X \) is a code with infinite deciphering delay over \( A \), then \( \alpha(X) \) is a code with infinite deciphering delay over \( C \).

Proof: By lemma 2.1, there exist \( x, \alpha(x) \in A^+ \) with \( x \alpha(x) \in X^+ \) and \( y, \alpha(y) \in X^+ \). By lemma 2.4, \( \alpha(X) \) is a code. Since \( \alpha \) is a morphism, then \( \alpha(x) \alpha(y) = \alpha(xy) \in \alpha(X)^+ \), \( \alpha(y) \alpha(z) = \alpha(yz) \in \alpha(X)^+ \) and \( \alpha(z) \alpha(y) \in \alpha(X)^+ \). Hence we have \( \alpha(x), \alpha(x) \alpha(y), \alpha(y) \alpha(z), \alpha(z) \alpha(y) \in \alpha(X)^+ \). Suppose \( \alpha(y) \in \alpha(X)^+ \). Then there exist \( x_i, K \in X \) such that \( \alpha(y) \in \alpha(y_i) K \alpha(y_n) = \alpha(y_i K y_n) \). Since \( \alpha \) is an injective morphism, then \( y = y_i K y_n \in X^+ \), which is a contradiction. Similarly, we have \( \alpha(z) \notin \alpha(X)^+ \). By lemma 2.1, \( \alpha(X) \) is a code with infinite deciphering delay. □

**Corollary 3.5** Let \( Z \subseteq A^+ \) and \( Y \subseteq B^+ \) be composable codes and \( X = Y \cup_{\alpha} Z \). If \( Y \) is a code with infinite deciphering delay, then \( X = \beta(Y) \) which is a code with infinite deciphering delay.

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4 Three-elements codes with infinite deciphering delay

We know that every two-element code is a code with finite deciphering delay (see [1]). In the following, we present two sufficient conditions for three-elements codes with infinite deciphering delay.

**Proposition 4.1** Let \( y, z \in A^+ \) and \( yz \neq zy \). If one of the following conditions holds:

1. \( \bar{x}y\bar{x} = zy \) and \( |z| < |\bar{x}y| \);
2. \( \bar{x} = xz \),

then \( X = \{\bar{x}, yz, zy\} \) is a code with infinite deciphering delay.

**Proof:** (1) If \( \bar{x}y\bar{x} = zy \) and \( |z| < |\bar{x}y| \), then \( z = \bar{x}y \), \( y = y_2\bar{x} \) and \( y = y_1y_2 \) for some \( y_1, y_2 \in A^+ \). Since \( y = y_1y_2 = y_2\bar{x} \), by lemma 2.5, then \( y_1 = uv \), \( y_2 = (uv)^ku \), \( \bar{x} = vu \) for some \( u, v \in A^+ \) and \( k \geq 0 \). Hence \( y = y_1y_2 = (uv)^{k+1}u \) and \( z = \bar{x}y_1 = vu^2v \). Since \( y \) and \( z \) are not powers of a common word, then \( u, v \in A^+ \). Suppose \( uv = vu \), then \( u^2v = vuv \). This implies \( yz = (uv)^{k+1}u^2v = (uv)^{k+2}uvu = vuv^2(uv)^ku = zy \), which contracts \( yz \neq zy \). So \( X = \{xy, yz, zy\} = \{vu(\bar{u})^{k+1}u, (uv)^{k+2}v, vu(\bar{u})^{k+2}u\} \). Then \( U_1 = \{vu\} \), \( U_2 = \{(uv)^{k+1}u, (uv)^{k+2}u\} \), \( U_3 = \{vu^2v, uv\} \), \( U_4 = \{(uv)^{k+1}u, (uv)^{k+2}u\} \). For any \( n \geq 3 \), we have \( U_{2n+1} = U_5 = \{vu^2v, vuvu^2v\} \) and \( U_{2n} = U_6 = \{(uv)^{k+1}u, (uv)^{k+2}u\} \). By lemma 2.3, \( X \) is a code. Let \( x = \bar{x}y\bar{x} = zy \in X^+ \). Then \( xy = (\bar{x}y)^2 \in X^+ \). Hence \( x, y, z \in X^+ \) and \( y, z \notin X^+ \). Thus \( X \) is a code with infinite deciphering delay by lemma 2.2.

(2) If \( \bar{x} = xz \), then \( X = \{\bar{x}, yz, zy\} = \{zy^2, yz, zy\} \). Since \( y \) and \( z \) are not powers of a common word, then \( U_1 = \{y\} \), \( U_2 = \{z\} \), \( U_3 = \{y, y^2\} \). For any \( n \geq 1 \), we have \( U_{2n+1} = U_3 \) and \( U_{2n} = U_2 \). By lemma 2.3, \( X \) is a code. Let \( x = zy \). Then \( x, y, z \in X^+ \) and \( y, z \notin X^+ \). Thus \( X \) has infinite deciphering delay. \( \square \)

**Example 4.2** Let \( x_1 = ba \), \( y_1 = aba \), \( z_1 = ba^2b \). Then \( \bar{x}_1y_1\bar{x}_1 = ba^2baba = z_1y_1 \). Thus \( X_1 = \{\bar{x}_1y_1, x_1z_1, z_1y_1\} = \{baaba, ababaab, baababa\} \), which is a code with infinite deciphering delay. Let \( \bar{x}_2 = ba \), \( y_2 = a \), \( z_2 = b \). Then \( \bar{x}_2 = z_2y_2 \). Hence we have \( X_2 = \{\bar{x}_2y_2, x_2z_2, z_2y_2\} = \{baa, ba, ab\} \) is a code with infinite deciphering delay. The prefix graphs of \( X_1 \) and \( X_2 \) are given in figure 2.

![Figure 2. The prefix graphs of \( X_1 \) and \( X_2 \)](image)
Theorem 4.3 Let $y, z \in A^*$ and $yz = zy$. If $x_i \bar{x} = \bar{y}y$ or $\bar{y}y = yz$ where $f$ is the primitive root of $y$ and $z$ and $x_i, f, f, x_i$, then $X = \{x_i, x_i \bar{x}, yz\}$ is a code with infinite deciphering delay.

Proof: (1) If $x_i \bar{x} = \bar{y}y$, by lemma 2.5, then $x_i = uv$, $\bar{x} = (uv)^k u $, $y = vu$ for some $u, v \in A^*$ and $k_i \geq 0$. Since $x_i$ and $f$ are incomparable for the prefix order, then $uv \neq vu$ and $u, v \in A^*$. Let $z = f^t$ for some $t \geq 1$. Hence $X = \{uv, (uv)^k u, kuv^t\}$. Then $U_i = \{(uv)^k u\}, U_2 = \{vu\}, U_1 = \{f^t\}$. Since $y = vu = f^t$, then $U_4 = \{vu\}$. For any $n \geq 1$, we have $U_{2n+1} = U_1$ and $U_{2n} = U_2$. By lemma 2.3, $X$ is a code. Let $x = x_i \bar{x} = \bar{y}y$. Then $x, y, yz, zy \in X^*$ and $y, z \notin X^*$. Thus $X$ has infinite deciphering delay.

(2) If $\bar{y}y = yz$, then there exist $u, v, 1 \in A^*$ and $k_2 \geq 0$ such that $\bar{x} = u_i v_i$, $y = (u_i v_i)^k i u_i$ and $z = v_i u_i$. Since $y$ and $z$ are powers of a common word, then $zy = yz$. It implies $(u_i v_i)^k i u_i = v_i u_i (u_i v_i)^k i u_i$. Thus $u_i v_i = v_i u_i$. We consider the following two cases.

(2-1) If $u_i, v_i \in A^*$, by lemma 2.5, $u_i$ and $v_i$ are powers of a common word. Therefore, $X = \{x_i, x_i u_i v_i, (u_i v_i)^k i u_i\}$. Let $y = (u_i v_i)^k i u_i = f^t$ for some $i \geq 1$. Then $u_i = f^t, v_i = f^t$ for some $r, s \geq 1$. Hence $X = \{x_i, x_i f^{r+s}, f^{(r+s)(k_2+1)}\}$. Since $x_i$ and $f$ are incomparable under the prefix order, then $U_i = \{f^{r+s}\}$ and $U_2 = \{f^{(r+s)(k_2+1)}\}$. For any $n \geq 1$, we have $U_{2n+1} = U_1$ and $U_{2n} = U_2$. By lemma 2.3, $X$ is a code. Let $x = x_i \bar{x} = \bar{y}y$. Then $x, y, yz, zy \in X^*$ and $y, z \notin X^*$. Thus $X$ has infinite deciphering delay.

(2-2) If $u_i = \epsilon$ or $v_i = \epsilon$, without loss generality, we let $u_i = \epsilon$. Then $x_i = v_i$, $y = v_i^k i$ and $z = v_i$. Since $y, z \neq \epsilon$, then $v_i \neq \epsilon$ and $k_2 \geq 1$. Hence $X = \{x_i, x_i v_i, v_i^k i\}$. Since $x_i$ and $f$ are incomparable for the prefix order, then $x_i$ and $v_i$ are incomparable for the prefix order. Therefore, $U_i = \{v_i\}$ and $U_2 = \{v_i^k i\}$ For any $n \geq 1$, we have $U_{2n+1} = U_1$ and $U_{2n} = U_2$. By lemma 2.3, $X$ is a code. Let $x = x_i v_i$. Then $x, y, yz, zy \in X^*$ and $y, z \notin X^*$. Thus $X$ has infinite deciphering delay. □

Example 4.4 Let $\bar{x} = a, y = ba, z = baba, x_i = ab$. Then $yz = zy$ and $x_i \bar{x} = \bar{y}y$. Then $X_3 = \{x_i, x_i \bar{x}, yz\} = \{ab, ab, (ba)^3\}$, which is a code with infinite deciphering delay. Let $\bar{x}' = (ab)^3, y' = ab, z' = (ab)^3, x_i' = ab$. Then $yy' = z'y'$. Therefore, $X_3 = \{x_i', x_i \bar{x}', yz\} = \{aa, ab(ab)^3, (ab)^4\}$, which is a code with infinite deciphering delay.

Example 4.5 Let $B = \{a, b, c\}$. Denote $x_i = ab$, $\bar{x} = c$, $y = z = c$. Then $\bar{x}y = yz = c^3$. By theorem 4.3, $Y$ is a code with infinite deciphering delay.

Let $A = \{0, 1\}$ and $Z = \{01010, 010101, 010100\}$. We define $\beta(a) = 01010, \beta(b) = 010101, \beta(c) = 0101001$. Then $X = Y \circ Z = \{0101010101, 010100101010001, 01010010101001, 0101001010101001\}$. 

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By corollary 3.5, \( X \) is a code with infinite deciphering delay. In fact, by example 3.3, \( Z \) is a code with infinite deciphering delay.

5 Conclusion

By example 4.5, we know \( Z \) is a three-elements code with infinite deciphering delay, but it doesn’t satisfies the conditions in theorem 4.1 and 4.3. In the future, we want to investigate the other constructions of three-elements codes with infinite deciphering delay.

References


