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A Note on the Efficiency of Indirect Taxes in an Asymmetric Cournot Oligopoly

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Abstract: Based on product homogeneity and Cournot competition, a recurrent finding in the literature is that ad valorem taxation is welfare superior to unit taxation in noncompetitive markets. This paper first observes that with asymmetric costs inefficient firms are more likely to be inactive in equilibrium under ad valorem taxation than under unit taxation. It is then illustrated that if the inefficient firms' unit costs and/or the ad valorem tax rate are high enough then unit taxation can be welfare superior to ad valorem taxation.

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1. Introduction

The recognition that unit (or specific) taxation and ad valorem taxation may lead to different outcomes under imperfect competition dates back to Cournot (1838) and Wicksell (1896). Suits and Musgrave (1953) were the first to show in a general monopoly setting that ad valorem taxation is welfare superior to unit taxation in that the former yields a larger total surplus than the latter generating the same tax revenue. In the ensuing fifty odd years since Suits and Musgrave (1953), many new developments have been advanced in regards to these two common forms of commodity taxation.[1] In particular, Delipalla and Keen (1992) and Anderson et al (2001) extend the classical result on the superiority of ad valorem taxation in the monopoly setting to Cournot oligopolies.[2] These results confirm the insight that price distortion is exacerbated less by an ad valorem tax than by a unit tax in imperfectly competitive markets. Although the literature mostly focused on firms with identical cost functions, Denicolo and Matteuzzi (2000) and Anderson et al (2001) show that this insight is still valid when firms have non-identical costs.[3]

[1] Keen (1998) provides a comprehensive review of both theoretical and empirical developments in the literature.

[2] Skeath and Trandel (1994) find that the stronger result that ad valorem taxation Pareto dominates (i.e., greater tax revenue, profit and consumer surplus) unit taxation always holds in the monopoly setting and holds true in a Cournot oligopoly when tax level is high.

[3] Wang and Zhao (2009) show that if products are differentiated and firms have non-identical costs unit taxation can be welfare superior to ad valorem taxation.

In this paper, we first observe that, in a homogenous good oligopoly, if firms have non-identical costs the least efficient firms are more likely to be inactive (producing zero output in equilibrium) under ad valorem taxation than under unit taxation when the inefficient firms' unit costs and/or the ad valorem tax rate are high enough. It will then be illustrated that, with more firms producing under unit taxation, it becomes possible for the total output to be greater under unit taxation and this larger output base makes it possible for unit taxation to be more efficient than ad valorem taxation in that the former can generate more tax revenue, higher consumer surplus and higher total surplus. The intuition for this result is as follows. Both imperfect competition and indirect taxation create price distortions. However, cost asymmetry can alleviate the welfare loss from these distortions since the more efficient firms will take up larger output shares and therefore lower the total cost of producing a given level of output, compared to when all firms are at the same (average) efficiency level. When all firms are active this role of cost asymmetry strengthens the superiority of ad valorem taxation. However, when the most inefficient firms are inactive under ad valorem taxation cost asymmetry will work to the advantage of unit taxation, so much so that the afore-mentioned welfare superiority result can be reversed.

2. The Basic Model

Consider an n-firm homogeneous good oligopoly with the inverse demand function p = f(Q), where p denotes price and Q denotes total industry output. As usual, we assume that f(Q) is decreasing in total output. Firm i has a constant unit cost of production given by c_i $(i = 1, \cdots, n)$. For convenience, we assume that $c_1 \leq c_2 \leq \cdots \leq c_n$ so firm 1 is the most efficient firm and firm n is the least efficient firm.

Firms compete in quantities à la Cournot. With a unit tax at the rate of t per unit of output, firm i's profit function is given by $\pi_i = f(Q)q_i - c_i q_i - t q_i$, where q_i denotes firm i's output and $Q = \sum_{i=1}^{n} q_i$. Assuming that there exists a unique Cournot equilibrium (or Cournot-Nash equilibrium) and all firms produce a positive quantity in equilibrium, the equilibrium output levels, denoted by q_i^u for firm i, satisfy the following first-order conditions: $f'(Q^u)q_i^u + f(Q^u) - c_i - t = 0, i = 1, \cdots, n$.[4] This condition stipulates that each firm equates its marginal revenue to its effective marginal cost (which is equal to its marginal cost plus the unit tax rate). It follows that firm i's equilibrium output is given by equation (1),

$$q_i^u = \frac{f(Q^u) - c_i - t}{-f'(Q^u)}, \qquad i = 1, \cdots, n \tag{1}$$

And, total industry output satisfies the equation (2):

$$Q^u = \frac{n[f(Q^u) - \bar{c} - t]}{-f'(Q^u)} \tag{2}$$

Where $\bar{c} = \sum_{i=1}^{n} c_i / n$ denotes the industry's average unit cost. It is obvious from equation (1) that, $q_1^u \geq \cdots \geq q_n^u$, namely firms' equilibrium outputs are ranked according to their efficiency levels with the most efficient firm 1 producing the most and the least efficient firm n producing the least.

With an ad valorem tax levied at the rate of τ fraction of gross revenue, firm i's profit function is $\pi_i = (1 - \tau)f(Q)q_i - c_i q_i$. Assuming that there exists a unique Cournot equilibrium and all n firms produce a positive quantity in equilibrium, the equilibrium output levels, denoted by

[4] Here and henceforth superscripts "u" and "a" denote equilibrium values under unit taxation and ad valorem taxation, respectively.

q_i^a for firm i, satisfy the following first-order conditions: $f'(Q^a)q_i^a + f(Q^a) - c_i/(1-\tau) = 0, i = 1, \cdots, n$. This condition stipulates that each firm equates its marginal revenue to its effective marginal cost (which is equal to its marginal cost adjusted by the ad valorem tax rate).[5] It follows that firm i's equilibrium output is given by

$$q_i^a = \frac{f(Q^a) - c_i/(1-\tau)}{-f'(Q^a)}, \qquad i = 1, \cdots, n \tag{3}$$

and total industry output satisfies the equation:

$$Q^a = \frac{n[f(Q^a) - \bar{c}/(1-\tau)]}{-f'(Q^a)} \tag{4}$$

From equation (3), $q_1^a \geq \cdots \geq q_n^a$.

Comparing equations (2) and (4), it follows immediately that if $\bar{c} + t = \bar{c}/(1-\tau)$ (or equivalently, $t = \tau\bar{c}/(1-\tau)$) then $Q^u = Q^a$. Applying these equations, subtracting (3) from (1) yields

$$q_i^u - q_i^a = \frac{\tau}{-f'(Q^u)(1-\tau)}(c_i - \bar{c}) \tag{5}$$

The following lemma is implied immediately by equation (5).

Lemma 1: If $t = \tau\bar{c}/(1-\tau)$ and all firms produce a positive output under either unit or ad valorem taxation then all firms with below average unit costs produce more under ad valorem taxation than under unit taxation and all firms with above average unit costs produce less under ad valorem taxation than under unit taxation.

This lemma indicates that the less efficient firms are more disadvantaged by an ad valorem tax than by a unit tax yielding the same level of total output. When total output is unchanged price is also the same and hence consumer surplus remains the same. As shown by Anderson et al (2001), total tax revenue is higher and total production cost is lower under ad valorem taxation. These facts imply that ad valorem taxation is more efficient than unit taxation (Proposition 2 of Anderson et al. (2001)).

A crucial condition for the above efficiency result is the assumption that all firms are active (i.e., producing a positive output) in equilibrium under either tax policy. The key observation for the point we try to make in this paper is that this condition may fail to hold if the ad valorem tax rate (τ) is high enough and/or the least efficient firms' unit costs are high enough.

Observation 1: If the ad valorem tax rate is sufficiently high and/or the least efficient firms' unit costs are sufficiently high then it can occur that the least efficient firms produce zero in equilibrium under ad valorem taxation while they produce positive outputs under unit taxation.

This observation is based directly on the fact that if c_n and/or τ are large enough then q_n^a in equation (3) is negative and $q_n^a < 0$ and $q_n^u > 0$ can hold simultaneously.

With more firms producing under unit taxation than under ad valorem taxation, it becomes easily possible for the total output to be greater under unit taxation and this larger output base also

[5] Note that firm i's profit function under ad valorem taxation can be rewritten as $\pi_i = (1-\tau)[f(Q) - ci/(1-\tau)qi$. Since maximizing πi is equivalent to maximizing $\pi i/(1-\tau)$ the firm's effective unit cost is $c_i/(1-\tau)$.

renders it likely that unit taxation generates more total tax revenue than that under ad valorem taxation. Both of these factors contribute directly to making unit taxation welfare superior to ad valorem taxation.

In the next section we work with a special market structure to demonstrate not only the above observation but also the main point of this paper that unit taxation can be more efficient than ad valorem taxation in that the former can generate more tax revenue, higher consumer surplus and higher total surplus.

3. A Homogenous Good Oligopoly with Two Types of Firms

In this section we consider an oligopoly with two types of firms. In this special case, we can provide explicit conditions for Observation 1 to hold. Moreover, for the case of linear demand, we obtain explicit equilibrium solutions and provide a simple numerical example to illustrate the main point of this paper.

There are m type I firms all having the constant unit cost c and k type II firms all with the constant unit cost $c+\delta$, where $c \geq 0$ and $\delta > 0$. Type I firms are the more efficient firms and the gap in cost efficiency between the two types of firms is measured by δ. Let $Q = g(p)$ denote the demand function (whose inverse is $p = f(Q)$) and $\eta(p) = -pg^{'}(p)/g(p)$ denote the price elasticity of demand. As usual, it is assumed that $\eta(p)$ is non-increasing in p. This assumption guarantees a unique Cournot equilibrium.

The following lemma (from Kamien et al (1992), pp. 487-488) presents the Cournot equilibrium for the oligopoly with two types of firms and no taxation. The lemma is stated with arbitrarily given unit costs c_1 and c_2 ($c_1 \leq c_2$) for the two types of firms.

Lemma 2:

(i) If $c_2 - c_1 < c_2/[m\eta(c_2)]$ then all firms produce a positive output in the Cournot equilibrium. Equilibrium output levels for the two types of firms are given by

$$q_1 = \frac{g(p)[c_1 + k(c_2 - c_1)\eta(p)]}{mc_1 + kc_2}, \quad q_2 = \frac{g(p)[c_2 - m(c_2 - c_1)\eta(p)]}{mc_1 + kc_2} \tag{6}$$

where the equilibrium price p satisfies the equation:

$$m + k - \frac{1}{\eta(p)} = \frac{mc_1 + kc_2}{p} \tag{7}$$

(ii) If $c_2 - c_1 \geq c_2/[m\eta(c_2)]$ then all type II firms produce a zero output in the Cournot equilibrium. Equilibrium output level for each type I firm is given by

$$q_1 = \frac{g(p)}{m} \tag{8}$$

where the equilibrium price p satisfies the equation:

$$m - \frac{1}{\eta(p)} = \frac{mc_1}{p} \tag{9}$$

Under a unit tax with rate t, unit costs for the two types of firms are $c_1 = c + t$ and $c_2 = c + \delta + t$. In this case, the condition in part (i) of Lemma 2 becomes

$$\delta < \frac{c+\delta+t}{m\eta(c+\delta+t)} \tag{10}$$

Under an ad valorem tax with rate τ, the effective unit costs for the two types of firms are $c_1 = c/(1-\tau)$ and $c_2 = (c+\delta)/(1-\tau)$. In this case, the condition in part (ii) of Lemma 2 becomes

$$\delta \geq \frac{c+\delta}{m\eta(\frac{c+\delta}{1-\tau})} \tag{11}$$

From Lemma 2, the following proposition is immediate.

Proposition 1: If the cost gap between the two types of firms satisfies conditions (10) and (11), then in the Cournot equilibrium with a unit tax at rate t all firms produce a positive output while in the Cournot equilibrium with an ad valorem tax at rate τ all type II firms produce a zero output.

The two inequalities (10) and (11) can obviously hold simultaneously. For example, if the equality $c+\delta+t = (c+\delta)/(1-\tau)$ holds for the parameters then the right-hand side of (10) is greater than the right-hand side of (11). Hence, the conditions in Proposition 1 are non-vacuous and Observation 1 is confirmed.

Observation 1 makes it possible for unit taxation to be welfare superior to ad valorem taxation. To provide explicit conditions for this possibility, we focus on the case of linear demand in the following. With linear demand, we can also provide a simple numerical illustration. Without loss of generality, let the linear demand equation be given by $p = a - Q$. Corollary 1 is a restatement of Proposition 1 for the linear case.

Corollary 1: With the linear demand function $p = a - Q$ and two types of firms, if

$$t < a - c - (m+1)\delta \quad \text{and} \tag{12}$$

$$\tau \geq [a - c - (m+1)\delta]/a \tag{13}$$

then all firms produce a positive output in the Cournot equilibrium with the unit tax t but only type I firms produce a positive output in the Cournot equilibrium with the ad valorem tax τ.[6]

This corollary places an upper bound on the unit tax rate (t) and a lower bound on the ad valorem tax rate (τ) such that in equilibrium all firms are active under unit taxation while only the more efficient type I firms are active under ad valorem taxation. Since (12) is equivalent to $\delta < (a-c-t)/(m+1)$ and (13) is equivalent to $\delta > [(1-\tau)a - c]/(m+1)$, conditions (12) and (13) alternatively place a lower bound and an upper bound on the gap (δ) in unit costs between the two types of firms.

Assuming that (12) holds, straightforward calculations using (6) and (7) give the equilibrium output for each type I and type II firm with the unit tax t as given respectively by

$$q_1^u = \frac{a-c-t+k\delta}{m+k+1}, \quad q_2^u = \frac{a-c-t-(m+1)\delta}{m+k+1} \tag{14}$$

and total industry output as given by

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[6] Note that, since the condition $\tau > [a - c - (m+1)\delta]/a$ is independent of the number of type II firms, it is not possible to have a corner solution in which a fraction of type II firms produce positive quantities.

$$Q^u = \frac{(m+k)(a-c-t)-k\delta}{m+k+1} \tag{15}$$

Assuming (13) holds. Then type I firms only produce positive quantities under ad valorem taxation. In this case, direct calculations using (8) and (9) give each type I firm's output as[7]

$$\hat{q}_1^a = \frac{a-\frac{c}{1-\tau}}{m+1} \tag{16}$$

and total industry output as

$$\hat{Q}^a = \frac{m(a-\frac{c}{1-\tau})}{m+1} \tag{17}$$

Based on these equilibrium expressions, we provide the following result that gives sufficient conditions for unit taxation to be more efficient than ad valorem taxation.[8]

Proposition 2: If in addition to (12) and (13) the following hold with at least one in strict inequality:

$$Q^u \geq \hat{Q}^a \tag{18}$$

$$tQ^u \geq \tau(a-\hat{Q}^a)\hat{Q}^a \tag{19}$$

$$(a-c-Q^u/2)Q^u - \delta k q_2^u \geq (a-c-\hat{Q}^a/2)\hat{Q}^a \tag{20}$$

where q_2^u, Q^u, and $\hat{Q}^a$ are respectively given by (14), (15) and (17), then the Cournot equilibrium under unit taxation with rate t welfare dominates the Cournot equilibrium under ad valorem taxation with rate τ.

Condition (18) implies directly that consumer surplus under unit taxation is greater than or equal to that under ad valorem taxation. Condition (19) simply states that tax revenue under unit taxation is greater than or equal to that under ad valorem taxation. The two sides in (20) correspond to total surplus (sum of consumer surplus, total profits and tax revenue) under the two tax schemes, respectively. Hence, condition (20) states that total surplus under unit taxation is greater than or equal to total surplus under ad valorem taxation.

In the above, we have focused on situations in which corner solution occurs under ad valorem taxation only. Obviously, our argument carries through even if corner solution occurs under both tax schemes as long as more firms are active in equilibrium under unit taxation than under ad valorem taxation.

4. Concluding Remarks

Based on product homogeneity and Cournot quantity competition, the prevailing literature has uniformly shown that ad valorem taxation is welfare superior to unit taxation in noncompetitive markets. This paper illustrates that in a Cournot oligopoly with asymmetric costs if the inefficient firms' unit costs and/or the ad valorem tax rate are high enough then unit taxation can be welfare

[7] The symbol "ˆ"denotes corner solution. We require $\tau < (a-c)/a$ so that type I firms are active under ad valorem taxation (i.e., $\hat{q}_1^a > 0$).

[8] The proof of the proposition is trivial and omitted. Conditions (18) and (19) are direct by definitions for consumer surplus and total tax revenue. Condition (20) involves simply the sum of consumer surplus, tax revenue and profits.

superior to ad valorem taxation. Although examples have been found in the context of Bertrand competition showing that unit taxation can be welfare superior to ad valorem taxation,[9] our result represents the first such finding in the context of Cournot competition.

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