Taylor Rule Revisited: from an Econometric Point of View¹

Claudia Kurz
University of Applied Sciences Mainz
Lucy-Hillebrand-Strasse 2, 55128 Mainz, Germany
Tel: + 49 6131 6283233 E-mail: claudia.kurz@wiwi.fh-mainz.de

Jeong-Ryeol Kurz-Kim (Correspondence author)
Deutsche Bundesbank
Wilhelm-Epstein-Strasse 14, 60431 Frankfurt am Main, Germany
Tel: +49 69 9566 4576 E-mail: jeong-ryeol.kurz-kim@bundesbank.de

Abstract: Based on a more realistic assumption, we modify the Taylor regression. The modified Taylor regression gives an explanation of why the (standard) Taylor regression is spurious (in the econometric sense, i.e. no stable relationship among the variables of interest) and, at the same time, a solution as to how central bank monetary policy can still be described by the Taylor rule. An empirical example using euro-area data confirms the compatibility of our modification with empirical data.

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1. Introduction

The well-known Taylor rule (Taylor, 1993) is given as

\[ i_t = r^* + \pi_t + a(\pi_t - \pi^*) + b y_t \] (1)

where \( i_t \) represents the central bank policy rate, \( r^* \) the equilibrium real interest rate, \( \pi_t \) the 12-month inflation rate, \( \pi^* \) the inflation target of the central bank, and \( y_t \) the output gap. According to the Taylor rule, the policy rate of the central bank is decomposed into response to deviations of current inflation from its target rate and response to deviations of the output gap from its target of zero.⁵ Taylor (1999) also estimated Equation (1) without arbitrarily setting \( r^* \) and \( \pi^* \) as

\[ i_t = c + a \pi_t + \beta y_t + u_t \] (2)

where \( c = r^* - a \pi^* \), \( a = 1 + a \), \( \beta = b \), and \( u_t \) is a disturbance. Our following econometric discussion will be based on this regression. One of the most important requirements for econometric analysis of the Taylor regression in (2) is that either all the three involved variables or the disturbance \( u_t \) must be stationary. The latter means that the three variables must be cointegrated if they are non-stationary. Otherwise, the regression (2) is a spurious one, causing the estimated parameter vector to be inconsistent and \( t \)- and \( F \)-statistics to diverge (see Phillips, 1986 for a more detailed discussion of this

¹ The views expressed in this paper are those of the authors and do not necessarily reflect the opinions of the Deutsche Bundesbank.

⁵ Taylor (1993) found that a rule with parameters set arbitrarily to \( r^* = 2 \), \( \pi^* = 2 \), \( a = 0.5 \) and \( \beta = 0.5 \) tracked the actual federal funds rate fairly well between 1987 and 1992.
In other words, any economic interpretation of the estimated parameters in the Taylor regression would not be allowed in the case of a spurious regression. The empirical results in the literature regarding the time series properties of the three variables are fairly mixed. Some authors just regard the variables as stationary (Clarida et al., 1998) and others document that, in general, \( i_t \) and \( \pi_t \) are non-stationary, while \( y_t \) is stationary. Moreover, some authors find no cointegration relationship between the three variables. See Gerlach-Kristen (2003) and Österholm (2006) for a more detailed discussion of this topic and the references therein.

In this paper, we introduce a modified version of Taylor regression based on a more realistic assumption described below. The modified Taylor regression provides an explanation of why the (standard) Taylor regression is spurious and, at the same time, a solution as to how central bank monetary policy can still be described by the Taylor rule. In other words, it combines the non-stationarity of the empirical data with the necessity of the existence of a cointegrating relationship for the economic theory. An empirical example using euro-area data demonstrates the compatibility of our modification with the empirical data.

2. The Modified Taylor Regression

The basic assumption for an econometric modeling of the Taylor rule is that a central bank monetary policy is based on a double mandate, i.e. it takes into consideration both inflation and economic activity in its monetary policy. This corresponds to the tradition of US monetary policy from which the Taylor rule stems. In the standard formulation of the Taylor rule (at least in the Taylor regression as an econometric model), therefore, it is assumed that the central bank always takes both inflation and economic activity into consideration with one and the same weight. This is not necessarily very realistic—at least from the viewpoint of European monetary policy; see footnote\(^3\) for a more detailed comment on this topic—and, hence, as will be shown, may cause the spurious problem. Therefore, we modify the standard Taylor regression by the following assumption:

**Assumption** The central bank monetary rates are a function of inflation when the current inflation is higher than the threshold inflation and a function of economic activity, otherwise:

\[
i_t = \begin{cases} 
  f(\pi_t; \bar{\pi}), & \text{if } \pi_t \geq \bar{\pi} \\
  g(y_t; \bar{\pi}), & \text{if } \pi_t < \bar{\pi}.
\end{cases}
\]

This assumption states that the central bank gives an absolute priority to stabilizing inflation and supports economic activity only when the inflation situation allows this.\(^2\) To capture this switching behavior by the central bank in its monetary policy as is assumed above, we incorporate an indicator function into the standard Taylor regression (2) as

\[
i_t = c + \alpha \pi_t l_{[\bar{\pi}, \infty)}(\pi_{t-1}) + \beta y_t \left[ 1 - l_{[\bar{\pi}, \infty)}(\pi_{t-1}) \right] + u_t
\]

where \( l_{[\bar{\pi}, \infty)}(\pi_{t-1}) \) is a usual indicator function, and 1, when \( \pi_{t-1} \geq \bar{\pi} \) and 0, otherwise. Because of unobservability of the current inflation rates, we use \( \pi_{t-1} \) in the indicator function for our modified Taylor regression. The lower bound in the indicator function, \( \bar{\pi} \), is probably a slightly higher positive

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\(^3\) This assumption is consistent with the earlier monetary policy tradition of the Bundesbank, which has been adopted by the European Central Bank. The tradition of the Bundesbank argues that the best and only way to contribute to economic growth is to guarantee price stability. Faust et al. (2001), however, find that, compared with the Bundesbank, the reaction function of the European Central Bank features a higher weight on the output gap relative to the weight on inflation.
number than the target inflation \( (\pi^*) \) and can be defined as a threshold inflation on which possible monetary policy support for economic activity depends. According to this modified formulation of the Taylor regression, the central bank concentrates on the reduction of inflation only when current inflation is higher than threshold inflation and has room for supporting economic activity when current inflation is lower than threshold inflation. From the econometric viewpoint (in the case that \( i_t \) and \( \pi_t \) are non-stationary, whereas \( y_t \) is stationary), this means that \( i_t \) and \( \pi_t \) will, or rather should be cointegrated when \( \pi_{t-1} \geq \bar{\pi} \). When \( \pi_{t-1} < \bar{\pi} \), the two variables are not cointegrated, but the non-stationarity of the \( \pi_t \) will not pass through into the residual process \( u_t \) because of the zero weight on inflation by construction. Therefore, the residual process of the Taylor modified regression will be still stationary if the monetary policy behavior of the central bank follows our assumption. In other words, if the assumption for our modification corresponds to the monetary policy behavior of the central bank and the macroeconomic variables included in the Taylor rule are non-stationary, then the standard Taylor regression cannot be cointegrated (although monetary policy is still a function of inflation and the output gap), because it can be cointegrated only with either \( \pi_t \) or \( y_t \) and the non-stationary part will pass through into the residual process. This is an explanation of why the standard Taylor regression is empirically often not cointegrated contrary to the implication of the economic hypothesis about the central bank’s behavior in its monetary policy.

### 3. Empirical Results

![Graph showing empirical data](image)

**Figure 1.** Empirical data

**Data:** The three variables for the euro area involved in the Taylor regression are three-month money market rates \( (i_t) \) as a measure of the central bank monetary rates, the 4\(^{th}\)-difference of the seasonally adjusted harmonized index of consumer prices \( (\pi_t) \) as a measure of the inflation rates and the output gap \( (y_t) \) constructed by calculating the percentage deviation of the seasonally adjusted real gross domestic product from its HP trend \( (\text{Hodrick and Prescott, 1997}) \) using a smoothing parameter \( \lambda = 1600 \). These are quarterly data covering the period from 1980 Q1 to 2007 Q4 (108 observations instead of 112 because of the 4\(^{th}\)-difference for constructing the inflation rates) and taken from the Bundesbank database. Figure 1 shows the
three variables, where the interest rates are marked as a dashed line (upper line), the inflation rates as a straight line (middle line) and the output gap as a straight line with a dot (lower line). The straight dotted line on 2.52 is, as will be explained, the estimated threshold inflation rate ($\hat{\pi}$).

Figure 1 contains — with respect to our modeling strategy — an important piece of visual information: the relationship between the interest rates and the output gap is weak when inflation rates are high and vice versa. Especially in the first one-third period in which the inflation rates are extremely high, the relationship between interest rates and the output gap seems to be very weak, while, in the other period, in which inflation rates are moderately high or low, the relationship between the two variables seems to become strong. This empirical fact supports the assumption for our modification, namely the central bank’s switching behavior in its monetary policy depending on the current inflation rates.

**Time series property of the data:** The ADF (Dickey and Fuller, 1979) test (assuming non-stationarity under the null) on the three variables gives test statistics of -1.85 (AIC=-1.62; lag=4); -1.72 (AIC=-2.77; lag=9); and -2.70 (AIC=-1.80; lag=1) for $i_t$, $\pi_t$ and $y_t$, respectively. Critical values are -1.95 and -2.60 at the significance levels of 95% and 99%, respectively. The KPSS (Kwiatkowski et al., 1992) test (assuming stationarity under the null) gives test statistics of 1.54 (Bartlett=4.57), 1.29 (Bartlett=3.62) and 0.11 (Bartlett=2.31) for $i_t$, $\pi_t$ and $y_t$, respectively. Critical values are 0.46 and 0.74 at the significance levels of 95% and 99%, respectively. The test results show, in line with many empirical results, that the interest rates and inflation rates can be seen as non-stationary, while the output gap is stationary.

**Choice for $\tilde{\pi}$:** Before the modified Taylor regression in (3) can be estimated, the threshold $\tilde{\pi}$ must be known. To determine a threshold, the standard likelihood-ratio (LR) technique is usually employed as considered in Tsay (1989). Similarly to the LR technique, we choose $\tilde{\pi}$ as

$$\hat{\pi} = \text{argmin}_{\hat{\pi}} \sum \sigma^2(\pi)$$

(4)

where $\pi = [x_{0.15n:n}; x_{0.85n:n}]$ with $x_{k:n}$ denoting the $k$-th order statistic from a sample of size $n$, meaning that 70% of the empirical density around the mean is used for detecting the threshold parameter. Moreover, in order to ensure a better fit of the model under the alternative of a threshold effect, we need a sufficient condition for determining the threshold value by the function in (4) as $\sigma^2(\pi) < \sigma_0^2$ with $\sigma_0^2$ being an estimated variance under the null. The estimated value of $\sigma_0^2$ is 2.52, where

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4 Andrews (1993) proposes a sample range of 70% as optimal to control the trade-off between zero power when $\sigma_{\min}^2 \to \sigma_{\max}^2$ and $\sigma_{\max}^2 \to \sigma_{\min}^2$ and zero detection of a possible threshold effect when $\sigma_{\min}^2, \sigma_{\max}^2 \to 0.5n:n$.

5 We perform a simulation study to check how well the detection technique works. For the data generating process, we use two tri-variate triangular cointegrated models (Phillips and Hansen, 1990) with the same residual process as follows:

$$i_t^{(0)} = c + \alpha \pi_t + \beta y_t + u_{1t}$$

$$i_t^{(1)} = c + \alpha \pi_t [\pi_t (\pi_t - \pi_t)] + \beta y_t [1 - I_{\pi_t (\pi_t - \pi_t)}] + u_{1t}$$

with the same exogenous (I(1))-variables for the both models, where, for our purpose (because the output gap is usually stationary variable), we use a stationary process for the second exogenous variable as $\pi_t = \pi_{t-1} + u_{2t}$; and $y_t = u_{3t}$, where $[u_{1t} u_{2t} u_{3t}] \sim \text{iid } N(0, \Sigma)$. We set $\alpha = \beta = 1; c = 0; \pi_t = \pi_{0.5n:n}$ without loss of generality. Furthermore, we set $M = [m_{ij}]$ with
\( \sigma^2(2.52) = 1.261 \), which is smaller than \( \sigma_0^2 = 1.265 \). The number of the inflation rates higher than the estimated \( \tilde{\pi} \) is 53 from the whole number of 108 observations. The value 2.52 as the estimated threshold (\( \tilde{\pi} \)) seems to be reliable from the viewpoint of the central bank’s empirical behavior. This finding of a threshold inflation supports the view that European monetary policy conforms to our modification of the standard Taylor rule.\(^6\)

Estimation and cointegration test: As shown in Figure 1, both the inflation rates and the interest rates have a negative trend for the whole sample period. To take these trends into consideration, we use a linear time trend term for our estimation\(^7\) as \( \tau = 10.7: -0.1 : 0.8 \). The estimated model without threshold parameter is given as

\[
i_t = 0.95 + 0.74\tau + 0.61\pi_t + 0.77y_t + \tilde{\pi}_t \\
(4.38) (20.1) (10.1) (5.68)
\]

The ADF test for cointegration (assuming no cointegration under the null) on \( \tilde{\pi}_t \) in (5) gives a test statistic of \( -2.24 \) (critical value \( -2.76 \) at 95\% and \( -2.45 \) at 90\% for \( T = 500 \); see Phillips and Ouliaris, 1990) and the KPSS test (assuming cointegration under the null) \( 0.35 \) (critical value \( 0.55 \) at 95\% and \( 0.32 \) at 90\% for \( T = 500 \); see Harris and Inder, 1994). To sum up, the results from the tests for integration and cointegration show that two of the three variables are integrated but the relationship is not cointegrated. The estimation using the pre-estimated threshold parameter, \( \tilde{\pi}_t = 2.52 \), results in

\[
i_t = 2.24 + 0.64\tau + 0.56\pi_t I_{(2.52,\infty)}(\pi_{t-1}) + 0.55y_t [1 - I_{(2.52,\infty)}(\pi_{t-1})] + \tilde{\pi}_t \\
(9.82) (19.1) (11.9) (2.54)
\]

The ADF test for cointegration on \( \tilde{\pi}_t \) in (6) gives a test statistic of \( -2.70 \) and the KPSS test \( 0.31 \). The results of cointegration test in the modified Taylor regression show that both of the tests give higher significance to a cointegrating relationship than that in the standard Taylor regression. Consequently, the relationship among the three variables in the modified Taylor regression can now be seen as cointegrated at the usual significance level, which results from the switching monetary policy behavior of the central bank.

### 4. Concluding Remarks

In this paper, we modified the Taylor regression by a more realistic assumption on the central bank’s behavior that the central bank gives an absolute priority to stabilizing inflation and supports economic activity only when the inflation situation allows this. This modified Taylor regression

\[
m_{ij} = 0.5 \text{ for all } i, j = 1, 2, 3; \text{ and } \Sigma = [\sigma_{ij}] \text{ with } \sigma_{ij} = 1 \text{ when } i = j \text{ and } 0, \text{ otherwise (the results for other designs of } M \text{ and } \Sigma \text{ are very similar to those presented). The results shows detection rates of } 71.97\%, 79.80\%, 91.71\% \text{ and } 95.74\% \text{ based on } 10,000 \text{ replications for } T = 50, 100, 250 \text{ and } 500, \text{ respectively.}
\]

\(^6\) As expected (because of the Fed’s double mandate), we find no threshold inflation in the US data covering the period from 1960 Q1 to 1999 Q4 used in Österholm (2005).

\(^7\) The estimation without the (negative) linear time trend term would give a negative relationship between \( i_t \) and \( y_t \) because of the negative trend in it, which stands in contrast to the economic theory.

\(^8\) The construction of the trend is arbitrary, but the estimation results are almost the same for other constructions.
enables us to give a possible explanation—at least for the euro-area data—for the alleged conflict between the economic theory (necessity of a stable relationship in the Taylor rule) and the usual empirical results (non-existence of a cointegrating relationship).

For further research, the next step would be to model a smooth transition (instead of a threshold transition) by which the central bank takes into consideration both inflation and economic activity simultaneously when current inflation is not very far from the target inflation. Other modifications using non-linear cointegration are possible as well.

References