Endogenous Growth, Efficiency Wages, and Persistent Unemployment

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Abstract: This paper establishes a theoretical relation between the level of unemployment and the economic rate of growth. In a model with a monopolistically competitive manufacturing sector and a competitive innovation sector, which both pay efficiency wages, the equilibrium unemployment rate – the Nawru – exhibits an unambiguously negative impact on the long-run growth performance, as it reduces the innovative capacity of the economy. Only if efficiency levels are different across sectors, a causal relation from the growth rate to the level of unemployment can be established, since less innovation shifts the burden to induce efficiency towards the manufacturing sector, thus fostering unemployment.

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1. Motivation

This paper investigates the common correlations and causalities between the level of unemployment and the economic growth rate. Some 30 years after Arthur Okun[8] postulated a relation between the change in the rate of unemployment and the change in the level of output, little theoretical and empirical work on the joint determinants of the level of unemployment and the rate of growth has been published. Evidence, therefore has to be collected from two distinct fields of research.

First, growth theory claims that changes in total factor productivity (TFP) are a key determinant for the long-run growth rate of an economy. Whether TFP is exogenous, in the sense of a Solow-residual, or endogenously determined by economic incentives to invest into human capital or research and development remains contested, but there is vast evidence that changes in TFP drive the economic growth rate[6].

Second, labour market theory claims that wages are set above the marginal product of labour in order to induce efficiency, thereby creating unemployment[2]. Shapiro and Stiglitz[16] present a dynamic efficiency model, but choose not to develop its growth implications. Wadhwani and Wall[18] provide evidence that firms apply efficiency considerations when setting wages.

By contrast, effects from TFP on unemployment as well as from efficiency on economic growth are rejected within the consensus of the theoretical literature. Layard, Nickell and Jackman[5] argue that unless there is complete insider power, changes in TFP do not alter the real wage, hence unemployment should remain unaffected. Salter[14] and Nickell and Kong[7] present weak favourable evidence for this hypothesis.

Theory also suggests that the growth rate should be independent of changes in labour efficiency. This is due to the Solow-condition, which states that at the margin the cost of an additional unit of
efficiency is equal to the cost of an additional unit of labour. Hence changes in efficiency would be offset by a reduction in employment, leaving output unaffected. This postulate is in sharp contrast with a vast literature that comes from the field of business administration, as for instance by Caves and Barton[3] for U.S. manufacturing, Oum and Yu[9] for the airline industry, and Sudit[17] for telecommunications. They find that firms that adopted methods to foster efficiency would not only see their stock-market value increase, but also reach significantly higher rates of growth.

The evidence, therefore, seems to be less opposing, if not even favourable of common sources that determine the level of unemployment and the rate of growth. Several theoretical papers have already investigated channels which may produce this result. In their seminal paper, Aghion and Howitt[1] find that changes in the rate of growth may alter the unemployment rate. Realizing that innovation driven growth models are models of structural change, where a proportion of the labour force has to seek new employment every period, time consuming search for work may cause persistent unemployment. In that respect endogenous growth will effect unemployment in three distinct ways. First there is the job creation effect, with new industries opening and hiring new workers. Then, there is the job destruction effect, as old firms leave the labour market. Then there is an effect, which Aghion and Howitt[1] loosely label the indirect effect. It indeed due to the fact that as the number of industries increases, firms can benefit less from scale effects, and production gets more labour intense.

However, the motive for searching is the possibility to find a better paid job lateron[11, 12, 13]. In Aghion and Howitt[1], all wages are identical, hence unemployment is not thoroughly argued. Schaik and de Groof[15] use efficiency wages to motivate wage differences. However, they motivate unemployment from searching, too, therefore obtaining a result similar to Aghion and Howitt[1], finding that growth may cause unemployment, but not vice-versa.

This paper finds an inverse relation, namely that unemployment causes lower growth rates. The model economy is populated by representative households, a monopolistically competitive manufacturing sector and a perfectly competitive innovation sector. Households maximise utility, whilst firms in both sectors maximise profits. Both manufacturers and innovators pay efficiency wages, although not necessarily at the same degree. Increasing internal efficiency, for instance by organisational changes, motivation, or changes in monitoring, implies a lesser use of incentives to motivate the workforce, hence the unemployment rate may decline. This leads to lower wage premia in manufacturing, thereby reducing unemployment, reducing marginal costs in the innovation sector, and increasing profit perspectives in manufacturing, which increases marginal revenues in the innovation sector. This will lead to more intense research and development, thereby fostering the rate of growth. A reduction in the long-run rate of unemployment may therefore foster economic growth.

2. The Household Problem

The model consists of households, firms and innovators. Households maximise inter-temporal lifetime utility, which under the assumption of log-linear felicity equals,

$$U_0 = \int_0^\infty e^{-\theta t} \ln c_t \, dt$$

(1)

where $c_t$ is consumption, and $\theta$ is the individual rate of time preference, subject to an inter-temporal budget constraint, which uses the consumption good as a numeraire,

$$\dot{a}_t = r_t a_t + w_t (1 - u_t) - c_t$$

(2)

which states that the change in wealth $a_t$ must equal the difference between consumption, interest income $r_t a_t$, and labour income, i.e. wages $w_t$ times employment $(1 - u_t)$. It is assumed that unemployment is equally distributed over the workforce, hence we may abstract from an unemployment insurance. Given that the unemployment rate is exogenous to the household decision...
problem, Hamiltonian optimisation with respect to consumption and wealth holdings yields the Keynes-Ramsey-Rule,

$$\frac{\dot{c}_t}{c_t} = r_t - \theta$$  \hspace{1cm} (3)

Consumption is assumed to consist of a bundle of specific consumption goods $x_{i,t}$, according to the following constant elasticity of substitution felicity function,

$$c_t = \left[ \int_0^{n_t} x_{i,t}^{\varepsilon - 1} \, di \right]^{\frac{1}{\varepsilon - 1}}$$  \hspace{1cm} (4)

Households maximise this felicity subject to the following budget constraint,

$$\int_0^{n_t} p_{i,t} x_{i,t} \, di \leq c_t$$  \hspace{1cm} (5)

Optimisation yields the demand function for a particular consumption good,

$$x_{i,t} = p_{i,t}^{\varepsilon} c_t$$  \hspace{1cm} (6)

where $\varepsilon$ is the price elasticity of demand, and a definition for the aggregate price index, which reads due to the normalisation,

$$\left[ \int_0^{n_t} p_{i,t}^{1-\varepsilon} \, di \right]^{\frac{1}{1-\varepsilon}} = 1$$  \hspace{1cm} (7)

### 3. Manufacturing

A particular consumption good is produced by a single firm, which exercises monopoly power on this particular market. Technology is given by,

$$x_{i,t} = e_{i,t} l_{i,t}$$  \hspace{1cm} (8)

where $l_{i,t}$ is the labour force employed in a particular firm, and $e_{i,t}$ is the efficiency of this workforce. Assume in accordance with Akerlof and Yellen\[2\] that both an increase in the relative wage vis-à-vis other firms, $\frac{w_{i,t}}{w_t}$, and an increase in the unemployment rate $u_t$ increase efficiency in the following specific form,

$$e_{i,t} = e^{1-\mu} \frac{w_{i,t}}{w_t}$$  \hspace{1cm} (9)

where $l_{i,t}$ is the labour force employed in a particular firm at the wage rate $w_{i,t}$, and the exponential term is the efficiency of this workforce. In accordance with Akerlof and Yellen\[2\], both an increase in the relative wage vis-à-vis the economy-wide average wage, $w_{i,t}/w_t$, and an increase in the unemployment rate $u_t$ increase efficiency. This particular functional form has an important feature, namely that the elasticity of the effort function with respect to any of its arguments is identical,

$$\sigma = \frac{\partial e_{i,t}}{\partial u_t} \frac{u_t}{e_{i,t}} = \frac{\partial e_{i,t}}{\partial w_{i,t}} \frac{w_{i,t}}{e_{i,t}} = \frac{\partial e_{i,t}}{\partial w_t} \frac{w_t}{e_{i,t}} = \frac{\partial e_{i,t}}{\partial \mu} \frac{\mu}{e_{i,t}} = \left( \frac{\mu}{w_t} \frac{w_{i,t}}{w_t} \right) > 0$$

This implies that in particular that a one percent decrease in unemployment will require a one percent increase in wages, holding labour costs thereby constant. This feature renders the solution tractable, and the functional form has been chosen for this reason. A more elaborate functional form with different elasticities with respect to its arguments would not change the qualitative results, as long as the monotonicity of the effort function (9), which is declining in $\sigma$ is preserved.

The downside of the simple functional form is that $\mu$ can be given three distinct interpretations.
The first interpretation of \( \mu \) is that it represents the degree at which a firm relies on firing to induce efficiency, or the degree at which workers perceive the threat of becoming unemployed. Assume for a moment that the exponent were equal to minus unity, which will be proven in equation (13) below, then workers will be willing to accept wages lower than average if and only if unemployment exceeds \( \mu \). As both the relative wage and the unemployment rate depend on economic conditions outside the firm, they shall be labelled “external efficiency”.

There is ample evidence that manufacturers can induce efficiency other than with a carrot (a high relative wage) and a stick (a high unemployment rate). Then \( \mu \) may be a function of the organisational structure (e.g. introducing internal controlling), or the motivation of the workforce (e.g. internal training, promotions, or seniority premia) within a manufacturing firm. As an increase in \( \mu \) reduces efficiency, \( \partial x_{i,t}/\partial \mu < 0 \), „internal efficiency“ may be measured by the index \( 1/\mu \). An interesting extension would be to analyze a functional form where “internal” and “external” efficiency can be distinguishable.

In this second interpretation of \( \mu \), it may reflect the extent at which the firm needs to rely on external efficiency, and \( 1/\mu \) the extent of internal efficiency. Whilst one can hardly verify the degree of motivation or the quality of organisation, the firing rate is easily accessible, hence firm owners (shareholders) may prefer efficiency inducing mechanisms that operate through wage premia, requesting high external efficiency, i.e. a high \( \mu \), and thus indirectly foster unemployment.

Profit maximisation in manufacturing implies that firms will increase their wage until the increase in effort is just offset by an alternative increase in employment, i.e. until the output elasticity of employment is equal to the output elasticity of the wage rate,

\[
\frac{\partial e_{i,t}}{\partial w_{i,t}} \frac{w_{i,t}}{e_{i,t}} = 1
\]

Any firm will increase (reduce) its wage relative to the average wage, whenever unemployment exceeds (lies below) \( \mu \). This will equi-proportionally increase (reduce) the average wage, thus inducing another round of relative wage increases (declines), implying ultimately that wages will increase without bound (decline to a zero rate). Hence a third interpretation of \( \mu \) is the manufacturing non-accelerating wage rate of unemployment (Nawru), as the specific functional form of the efficiency function reduces equation (10) to,

\[
\mu = \frac{w_{i,t}}{u_t}
\]

The efficiency condition (10) implies that productivity in manufacturing will equal unity,

\[
e_{i,t} = 1
\]

When setting prices, firms maximise profits subject to demand (6) and technology (9), which yields the following first order condition,

\[
p_{i,t} = \frac{\varepsilon w_{i,t}}{\varepsilon - 1 e_{i,t}}
\]

stating that the price the firm charges equals the mark-up over costs, the wage in efficiency terms. Note that the manufacturing sector is completely symmetric, as every firm will choose identical efficiency levels due to equation (12), set identical wages due to equation (9), and identical prices due to equation (13). The demand function (6) then implies that output will be identical for all manufacturers, and therefore also employment, due to the production function (8). This allows us to identify economic profits of the manufacturing sector, \( d_o \), by simple aggregation over all individual profits,
\[ d_t = n_t \pi_{i,t} = n_t \left( p_{i,t} x_{i,t} - w_{i,t} l_{i,t} \right) = \frac{1}{\varepsilon - 1} n_t w_{i,t} l_{i,t} \]  

where \( i \) now represents any representative manufacturer.

### 4. The Innovation Sector

It takes time, effort, knowledge and luck to succeed in innovation. We account for all these factors when modelling the innovation of new varieties according to the following aggregate technology,

\[ n_t = \phi n_t e_t l_t \]  

where \( \phi \) is productivity in innovation and measures luck, \( l_t \) is the labour force employed in the R & D sector and together with \( e_t \), (to be defined below) measures effort, and \( n_t \) is an externality, stating that it is easier to innovate when the stock of knowledge, i.e. the existing number of innovations, is large. Efficiency is defined according to equation (9), where we do assume a rather plausible difference in internal efficiency between the two sectors,

\[ e_t = e^{-\delta u / u_t / w_t} \]  

This condition differs from manufacturing efficiency (9) by \( \delta \). If \( \delta \) is less than unity, then ceteris paribus employees in the innovation sector are less productive than workers in manufacturing. This is equivalent to stating that the exogenous firing probability of standard efficiency wage models is lower in the innovation sector, which seems to fit the facts. Nonetheless, we shall consider all potential cases of \( \delta > 0 \) in the following.

In the innovation sector, the efficiency relation may of course be labelled creativity. An increase in unemployment need not un-ambiguously raise creativity. Whilst an initial increase in unemployment will raise creativity here as well, very high rates of unemployment may result in the opposite effect. Given risk averse researchers, they may engage in less risky, short-sighted projects with a lower yield, thus reducing the value of output of the innovation sector, or measured efficiency (16). Approximating efficiency (16) by a quadratic efficiency equation of the form \( \exp\left[1 - \mu w / s_t (\delta - u_t)^2\right] \), the qualitative results as presented in the following chapters will not change for peaks \( \delta \) in the efficiency function between zero and unity. (16) therefore represents a good linear approximation to the more general case.

In line with most endogenous growth models\(^{[4, 13]}\), it is assumed that the search for profit fosters innovations. Maximising profits of an innovator subject to technology (15) and efficiency (16), we find that the salaries set by research institution will be above market clearing, following the Solow condition,

\[ \frac{s_t}{e_t} \frac{\partial e_t}{\partial \delta e_t} = 1 = \frac{\delta u}{u_t} / \frac{s_t}{w_t} \]  

Competitive firms in the innovation sector will invest into the development of new products until the marginal revenues just offset marginal costs. Assuming that labour is the only input in R & D, marginal costs equal salaries \( s_t \). Assuming perfect competition in the innovation sector, the price of a new innovation \( q_t \) will equal,

\[ q_t = \frac{s_t}{\phi e_t n_t} \]
Potential manufacturers will at most pay a price for a novel innovation equal to the discounted stream of profits. No arbitrage on capital markets implies that an investor should be indifferent between a risk free investment of the amount \( q_t \), yielding interest payments of \( r q_t \), or the purchase of a manufacturer’s stock. From the later, she may benefit from changes in the stock’s value over time, and profits distributed to shareholders,

\[
\hat{q}_t + \pi_{t,t} = r q_t \tag{19}
\]

By the symmetry of the model, the value of all firms in manufacturing, or the stock market capitalisation \( v_t \), will equal \( n q_t \). Aggregation over all firms, and substitution of dividends \( d_t \) from equation (14), the firm value from the innovator’s first order condition (18), and the interest rate from the inter-temporal Euler equation (3), yields,

\[
\frac{\dot{v}_t}{v_t} = \frac{\dot{n}_t}{n_t} + \frac{\phi n_t l_{t,t}}{\delta (\varepsilon - 1)} = \frac{\dot{c}_t}{c_t} + \theta \tag{20}
\]

5. The Labour Market

Normalising the labour force to unity, we find that employees will either work in manufacturing or research, hence the labour market clearing condition reads,

\[
\int_0^n l_{i,t} di + l_t = 1 - u_t \tag{21}
\]

Note that average wages \( w_t \) are defined as wages paid by firm \( i \) to its workers \( l_{i,t} \), and salaries \( s_t \) by research institutions to its employees \( l_t \), divided by the total labour force,

\[
w_t = \left[ s_t l_t + \int_0^n w_{i,t} l_{i,t} di \right] / \left[ l_t + \int_0^n l_{i,t} di + u_t \right] \tag{22}
\]

The average wage includes income from employment and unemployment, where the later has been set to zero. Moreover, as leisure does not enter the utility function, opportunity costs of unemployment are simply wages foregone. Hence unemployed appears in the denominator, but not in the numerator.

6. Equilibrium employment

The wage equation (22) may be simplified, applying the two Solow-conditions (11) and (17), and the labour market clearing condition (21), to reduce equation (22) to,

\[
\mu (\delta - 1) l_t = (1 + \mu) u_t - \mu \tag{23}
\]

Assuming for an instant that both manufacturers and innovators share the same degree of internal efficiency, or equivalently, that \( \delta = 1 \), then equation (23) reduces to

\[
u_t = \frac{\mu}{1 + \mu} \tag{24}
\]

Given identical degrees of internal efficiency, the unemployment rate is independent of any other economic parameters, and in particular independent of the economic growth rate. Note that the average rate of unemployment \( u_t \) is lower than the non-accelerating wage rate of unemployment \( \mu_t \), or the median rate of unemployment, due to the concavity of the wage index. Integrating the budget constraint (2), we find that stock market capitalisation and consumption grow at the same rate, reducing the no-arbitrage-condition (20) by substitution of the labour market clearing condition (21).
and innovation technology (15) to,
\[(\delta \epsilon + 1 - \delta) l_t = 1 - u_t + \frac{\theta}{\phi} (1 - \delta)\]  
(25)

Eliminating innovation sector employment by equations (23) and (25), the equilibrium rate of unemployment in the model emerges to be,
\[u_t = \frac{\mu (\phi \delta \epsilon - \theta)}{\phi \delta \epsilon (1 + \mu) + \phi (1 - \delta)}\]  
(26)

Whilst the denominator of this expression is strictly positive by definition, the numerator is greater than zero if and only if \(\phi \epsilon \delta \geq \theta\). Given the fact that the rate of time preference \(\theta\) is typically small, and productivity in the innovation sector, \(\phi\), is not too different from manufacturing, where it equals unity by definition, this condition is likely to hold.

Equation (26) implies that an increase in patience reduces unemployment. The intuition is that more patient societies are more willing to defer consumption and invest into innovations that yield higher welfare later on. This leads to an increase in the innovation sector labour force, reducing average wages and thus leading to higher aggregate employment. Similarly, an increase in research productivity reduces unemployment, as the lower relative wage in the innovation sector will enable innovators to hire a larger share of the labour force.

An increase in the price elasticity of product demand, \(\epsilon\), will unambiguously reduce unemployment. Here, the reason is that a reduction of the imperfect market externality will lead to higher levels of output and production. Evidently, an increase in the non-wage accelerating rate of unemployment will reduce the unemployment rate, for evident reasons. Finally, an increase in the firing probability in innovation, \(\delta\), will reduce unemployment if and only if \(\phi \epsilon < \theta (1 + \mu) - \theta\).

Summarising, we find that patience, high research productivity, product substitutability, and high internal efficiency, \(1/\mu\), are beneficial for employment, whilst the effect of the firing probability in innovation remains ambiguous.

By substituting unemployment (26) back into equation (23), we can obtain the share of workers in the research sector,
\[l_t = \frac{1 + \mu}{\mu (\delta - 1)} \frac{\phi \delta \epsilon - \theta}{\phi \delta \epsilon (1 + \mu) + \phi (1 - \delta)} - \mu\]  
(27)

which is constant as well. We can use the labour market clearing condition (21) to identify the number of workers in the manufacturing sector,
\[\int_0^n l_{t,\epsilon} d\epsilon = 1 + \mu - \frac{1 + \mu \delta}{\delta - 1} \frac{\phi \delta \epsilon - \theta}{\phi \delta \epsilon (1 + \mu) + \phi (1 - \delta)}\]  
(28)

Note that employment in the manufacturing sector is constant, too, but with an increasing number of varieties the number of workers in each firm \(n\), is decreasing.

7. Equilibrium growth

Equations (23) and (25) may also be solved for innovation sector employment, which will be constant. Given a constant unemployment rate from equation (26), we find from the definition of the consumption basket (4), manufacturing technology (8), and the labour market clearing condition (21), that consumption growth is \(1/(\epsilon - 1)\) times the growth rate of innovations. Hence, the growth rate of the economy can be derived to give,
\[
\frac{\dot{c}_t}{c_t} = \frac{\phi - (1 + \mu)\theta}{\delta c(\varepsilon - 1)(1 + \mu) + (\varepsilon - 1)(1 - \delta)}
\]

which is positive if and only if \( \phi > (1 + \mu)\theta \). This condition is met when innovation productivity does not fall too far behind manufacturing productivity. There are no transitional dynamics, so that the economy will immediately jump to the equilibrium growth path.

The economy will grow faster if consumers get more patient, as they will allocate more funds to the innovation sector, leading to a higher output of innovations and a higher output growth rate. Similarly, higher productivity in the innovation sector will also foster economic growth, whilst a higher firing probability in the innovation sector, \( \delta \), will reduce growth, as it reduces innovation sector productivity. Whilst an increase in product substitutability reduced unemployment, it will be detrimental to economic growth, unless \((1 + \mu)(2\varepsilon - 1) > (\delta - 1)/\delta \), which certainly holds for \( \delta < 1 \), but for most realistically positive values of \( \delta \) as well. The intuition is that high substitutability reduces monopoly rents, thus reducing the incentive to engage in innovations, leading to a decline in output growth.

Finally, an increase in the non-accelerating wage rate of unemployment, \( \mu \), will reduce economic growth if and only if \( \theta(\delta - 1) < \delta \phi \). This condition will hold whenever \( \delta \) is less than unity, which is the more plausible case. Even if \( \delta \) exceeds unity, a sufficient condition for a negative impact of the level of unemployment on the rate of economic growth is \( \delta > 1/(\varepsilon - 1) \). This is also equivalent to stating that the firing probability in innovation has a negative impact on unemployment.

Summarising, we find that patience, high innovation productivity, a low firing probability in the innovation sector, low product substitutability, and high internal efficiency or a low Nawru, all foster economic growth. Most importantly, we have established a typically negative trade-off between the level of unemployment, to be more specific the unemployment rate, and the change in output, or the economic rate of growth.

Defining output as the sum of all intermediate inputs, \( y_t = n_t x_t \), we find that output growth is zero, or that economic growth is only due the improvement in varieties, generally labelled “intensive growth” in the literature. Another way to see this point is to take logs and derivatives of the demand function (6) to find that the growth rate of prices equals consumption growth (29), or that consumers are willing to pay an ever higher price for inputs as they become more and more valuable with an increase in variety. Taking logs and derivatives of price setting (13), we find that real wage growth equals,

\[
\frac{\dot{w}_t}{w_t} = \frac{\phi - (1 + \mu)\theta}{\delta c^2(1 + \mu) + \varepsilon(1 - \delta)}
\]

which is less than consumption growth, implying that profits increase with an increase in variety.

### 8. Conclusions

The paper has established a relation between the unemployment rate and the long-run rate of economic growth. The main result is that an increase in the unemployment rate, caused by a greater pressure towards outside efficiency, will also reduce the growth rate of the economy, as employment will be both reduced in the manufacturing and the innovation sector, the later leading to a decline in the innovation rate, and hence slower economic growth.

Only if internal efficiency considerations are different between the manufacturing and the innovation sector, which may be due to differences in the ease to monitor worker’s effort, will there also be a channel that leads from the growth rate to unemployment. If patience in the economy declines, if manufacturing products get more homogenous, or simply if innovation productivity...
declines, the economy will accumulate less innovations, thereby reducing the rate of economic growth. This will also lead to a shift away from innovation sector employees towards manufacturing workers. As the manufacturing sector now receives a greater weight in average wages, wage increases in manufacturing must increase to induce the desired level of efficiency, thereby fostering unemployment.

References